Learning Curves

Lunch & Learn

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Presented by:
Judd Fussell, PhD
Defense Acquisition University
Judd.Fussell@DAU.MIL
(256) 922-8757

Objectives

- Origin of Learning Curve Theory
- Areas/Conditions for learning
- Summarize basic learning curve theory
  - Unit Formulation
  - CUMAVE Formulation
- Production Breaks/Lost Learning

Introduction

- **Background/Origin**
  - **T.P. Wright (1936)**
    - **Cumulative average costs decreases at a constant rate**
  - **J.R. Crawford (Post-WWII)**
    - **Unit costs decrease at a constant rate**
- **Different Names**
  - **Learning Curve**
  - **Cost Improvement Curve**
  - **Experience Curve**
- **Used for calculating Production Costs**
  - **Industry Standard in both commercial and non-commercial (government) applications**
- **Basic Models**
  - **Unit Theory and Cumulative Average**
  - **Anderlohr and Retrograde (Production Breaks)**

Cost Improvement

- **Associated with Repetitive Actions**
- **Recurring/Variable Costs**
- **Direct result of quantity of units**
- **Elements of Manufacturing**
  - **Material**
  - **Tooling**
  - **Fabrication**
  - **Assembly**

Why do we see cost improvement?

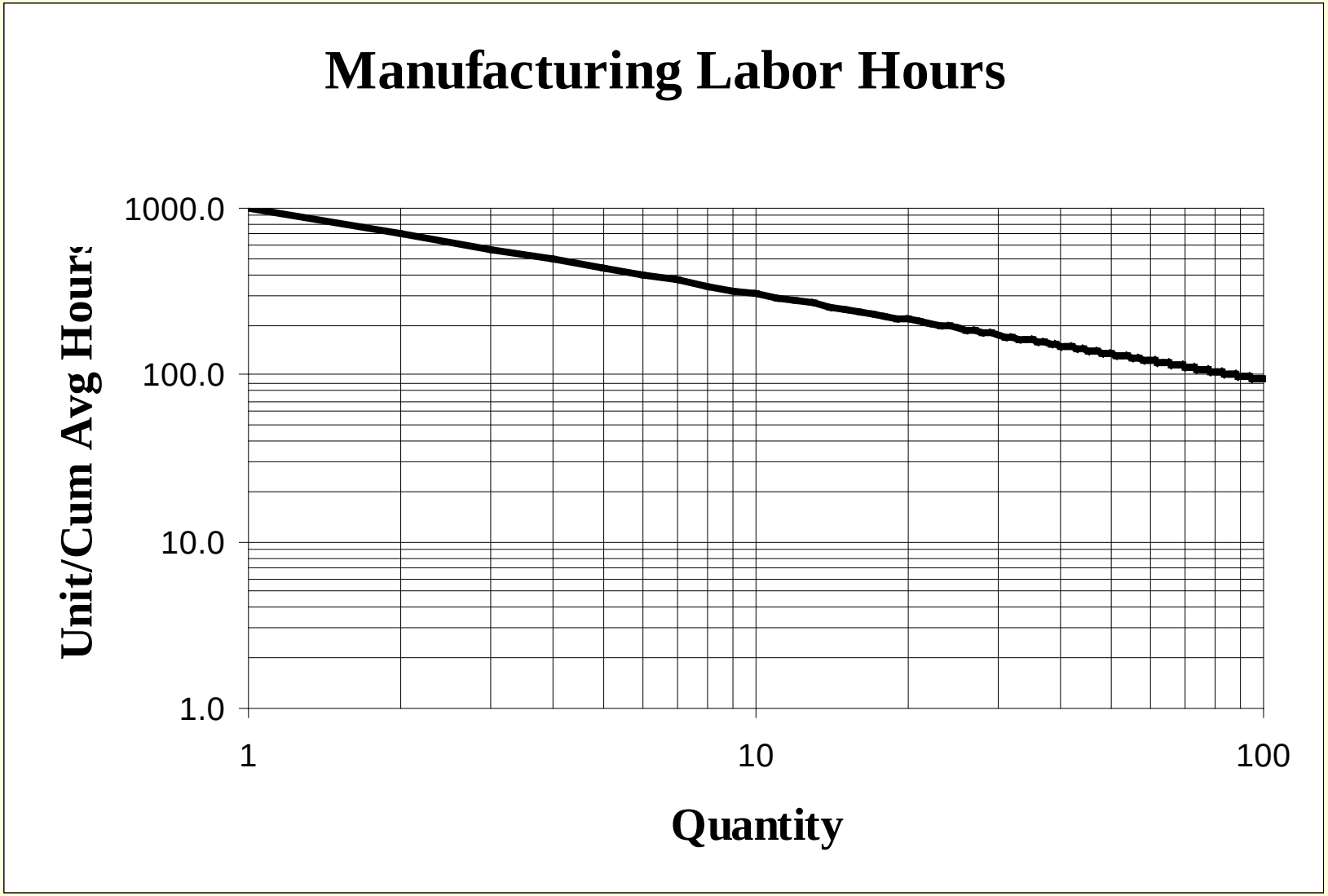
- **Worker Learning**
- **Supervisor Learning**
  - **Resource use**
  - **Work Environment**
  - **Motivation**
- **Process Improvement**
  - **Optimization of production line/workflow**
  - **Better understanding of instructions/drawings**
  - **Better Tooling**
  - **Better Design**
- **Organizational Changes**
  - **Inventory Practices and Vendor Relationships**
  - **Decentralization or Centralization of tasks**

Conditions that facilitate cost improvement

- **More complex tasks/end items have more opportunity for improvement; cost improvement curve not normally applied to simple end items**
- **Limited or minimized changes**
- **Continuous, uninterrupted manufacturing process**
- **Motivation/Pressure to improve**
  - **Competition**
  - **Profit**

Cost Improvement Curve

Transformed "Curve"



Learning Curve Slope

- **Slope defined as the change in Y given a change in X**
- **In *log* (or *ln*) space, the slope equates to a constant percent change in Y**
- **Learning curve theory states that as the total quantity of units doubles, the cost decreases by a constant percent**
- **This decrease is called the Rate of Learning (ROL)**
- **1.00 – ROL = Learning Curve Slope**

$$\text{Cost}_{2x} = \text{Cost}_{x}\ (\text{Learning Curve Slope})$$

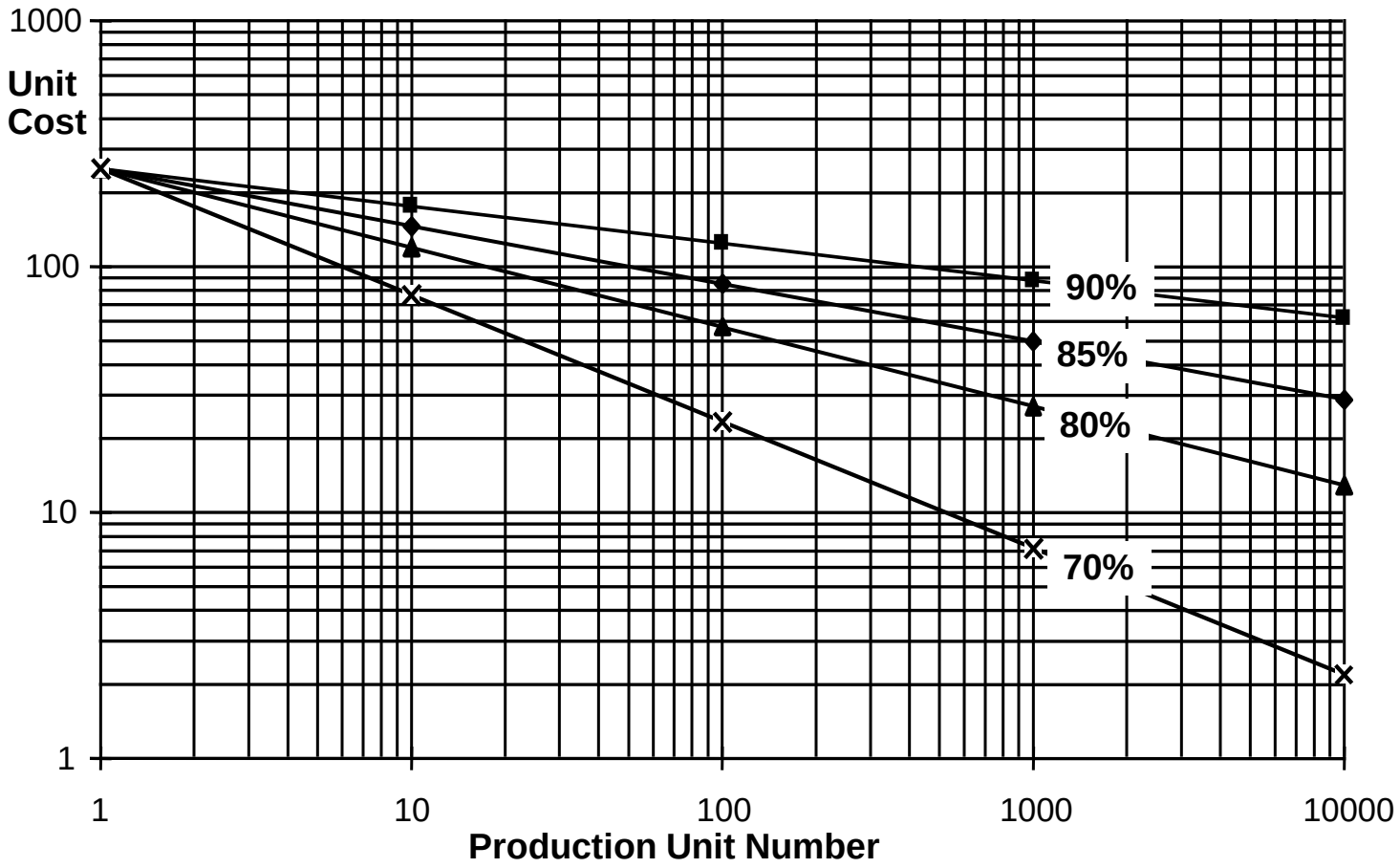
Example

Assume a 90 % LEARNING CURVE

Unit #	Direct Labor Hrs	
1	100	x 90% = 90
2	90	x 90% = 81
3	85	
4	81	x 90% = 73
5	78	
6	76	
7	74	
8	73	
16	66	
32	59	

What is the Rate of Learning (ROL)?

Different Slopes



Cost Improvement Curve Theory Applications

- **Unit:**
  - **sufficient time for production planning**
  - **hard tooling**
  - **stable design**
- **Cumulative Average:**
  - **insufficient time for production planning; sometimes concurrent with development**
  - **soft tooling**
  - **design instability**
- **Rate, Fixed Cost, etc which are variations of unit learning (Not covered today)**

Unit vs Cum Average

- **No firm rules but consider:**
- **Analogous Systems**
  - **What theory has worked best on them**
- **Industry Standards**
- **Historical Experience**
  - **Contractor choice**
  - **Estimator choice**
- **Expected Production Environment**
  - **Cum theory best in “soft” or prototype tooling, inadequated supplier base, early design changes expected, subject to short lead times**
  - **Unit theory best where contractor is well prepared in terms of tooling, suppliers, lead-times, etc.**

Unit Theory

- **Definition: As the total quantity of units doubles, the <u>unit cost</u> decreases by a constant percent**

- **Formula: $Y_x = AX^b$**

  where: Y_x - Cost of a specific Unit

  A - Cost of the first unit (aka T1)

  X - Unit number under consideration

  b - Slope Coefficient = LN(slope)/LN(2)

Unit Formulation

Define the Learning Curve Equation:

Unit #	Hours/Unit
1	917.00
2	779.45
4	662.53
8	563.15
16	478.68

$$Y_x = AX^b$$

Unit Formulation

Define the Learning Curve Equation:

Unit #	Hours/Unit	
1	917.00	
		.85 = 85%
2	779.45	
		.85 = 85%
4	662.53	
		.85 = 85%
8	563.15	
		.85 = 85%
16	478.68	

A = Cost of the First Unit = 917.00

b = Slope Coefficient = LN(slope)/LN(2)

= LN(.85)/LN(2) = -0.1625/0.6931 = -0.2345

$$Y_x = AX^b = 917.00X^{-0.2345}$$

Unit Calculations

Calculate the cost of Unit 100:

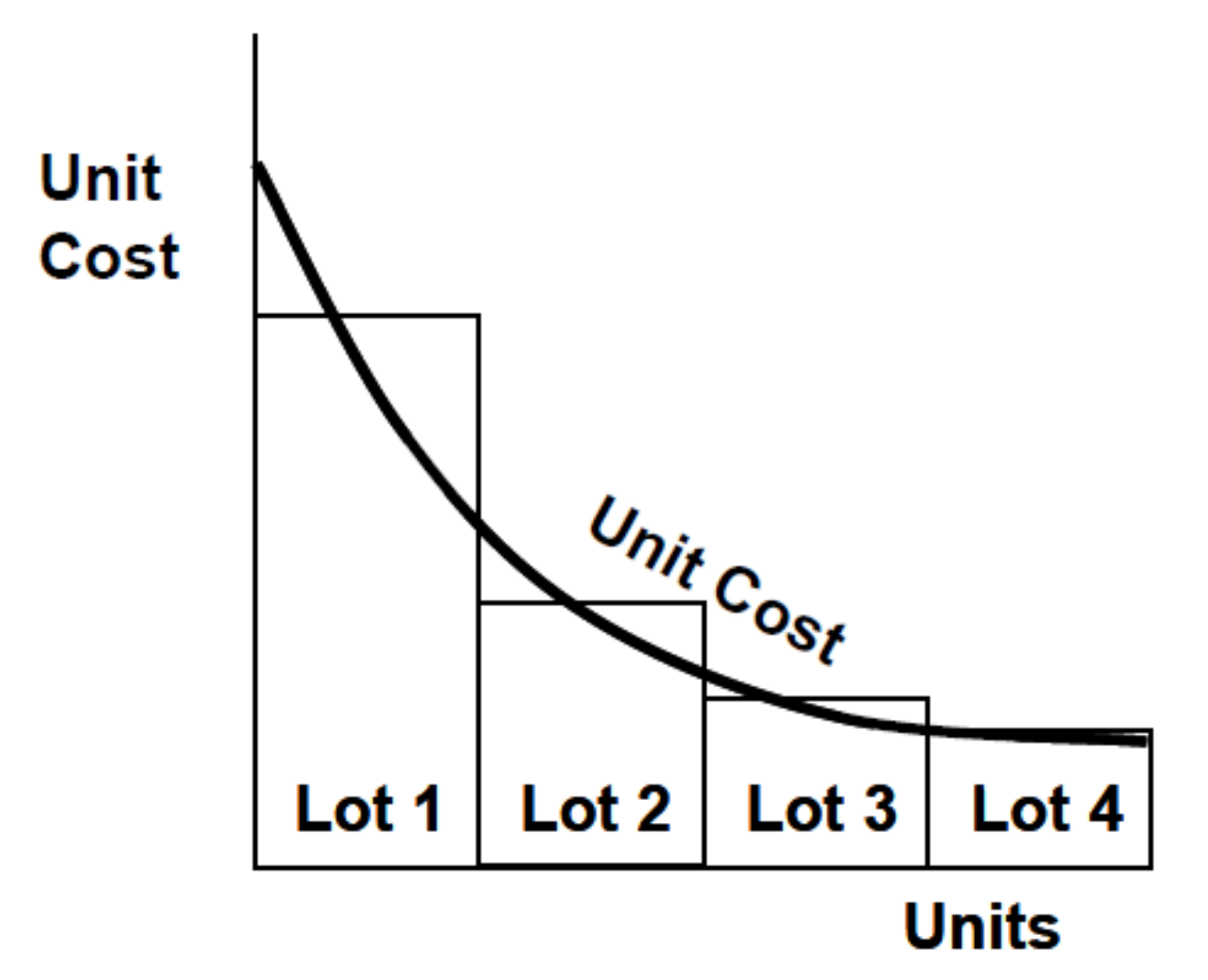
$$Y_x = AX^b = 917.00X^{-0.2345} =$$

$$Y_x = AX^b = 917.00X^{-0.2345} = 917\ (100)^{-0.2345}$$
$$= 311.49$$

Lot Data

- **We normally get production cost information**
  - **In lots**
  - **Total Cost of Lot**
  - **Number of units in Lot**
- **Since we don't normally get a cost of the first unit or unit costs to derive a slope using regression, how can we get these values so we can use our equations?**

Lot Midpoint (ALM)



Algebraic Lot Midpoint (ALM)

- **Algebraic Lot Midpoint (ALM) or Lot Midpoint (LMP)**
  - **The ALM is defined as the theoretical unit whose cost is equal to the Average Unit Cost (AUC) or Lot Average Cost (LAC) for that lot on the learning curve.**
- **The ALM value represents the X value and the AUC represents the Y value when using lot data**

AUC = Lot Cost / Lot Size

Lot Size = L – F + 1
F = 1st unit # in lot
L = last unit # in lot

How to arrive at Lot Midpoint

Again, we have two options on how to come up with the ALM (or LMP)

1. **Use Learning Curve Software**
2. **Approximate the ALM using these rules:**

For FIRST Lot:

If lot Size < 10, ALM = Lot Size / 2

If lot Size ≥ 10, ALM = Lot Size / 3

For ALL OTHER Lots:

$$ALM = \frac{F + L + 2\sqrt{FL}}{4}$$

F = 1st unit # in lot
L = last unit # in lot

Let's review a bit

The basic equation for Unit Theory is:

$$Y = AX^b$$

This can be transformed into a linear equation as follows:

$$\ln Y = \ln A + b \ln X$$

Example

Lot	Units	Lot Size	Cost	ALM (x)	AUC (y)
1	1-15		36750		
2	16-25		19000		
3	26-85		90000		
4	86-115		39000		
5	116-165		60000		

Calculating Slope and Intercept

- **Load Excel and using the data on the previous slide**
- **Construct a spreadsheet to calculate the ALM (x values) and the AUC (y values)**
- **After you have these values add an additional two columns and take the natural log of each of these LN(ALM), LN(AUC)**
- **Run a regression analysis on the LN values using LN(AUC) for the dependent variable,Y, and LN(ALM) for the independent variable, X**
- **Write down the equation you get**
- **Convert to learning curve equation**

Excel Example

1	2	3	4	5	6	7	8
Given	Given		Given				
Lot	**Units**	**Lot Size**	**Cost**	**ALM (x)**	**AUC (y)**	**LN (x)**	**LN (y)**
1	1-15	15	36750	5	2450	1.6094	7.8038
2	16-25	10	19000	20.25	1900	3.0082	7.5496
3	26-85	60	90000	51.26	1500	3.9368	7.3132
4	86-115	30	39000	99.97	1300	4.6049	7.1701
5	116-165	50	60000	139.42	1200	4.9375	7.0901

Excel Linear Regression

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.997930225
R Square	0.995864734
Adjusted R Squ	0.994486312
Standard Error	0.02167507
Observations	5

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.339421	0.339421	722.4672	0.000113001
Residual	3	0.001409	0.00047		
Total	4	0.340831			

	Coefficients	*tandard Err*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*ower 95.0%*	*Upper 95.0%*
Intercept	8.169965733	0.030757	265.6256	1.18E-07	8.072081726	8.26785	8.072082	8.26785
X Variable 1	-0.216775911	0.008065	-26.87875	0.000113	-0.242442221	-0.19111	-0.242442	-0.19111

What is the Linear Equation?

Y = 8.1700 - 0.21678X

Example

Lot	Units	Lot Size	Cost	ALM (x)	AUC (y)	LN (x)	LN (y)
1	1-15	15	36750	5	2450	1.6094	7.8038
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5	116-165	50	60000	139.42	1200	4.9375	7.0901

Y = 8.1700 - 0.21678X…this is really LN(y) = LN(A) + bLN(x)

To convert this to $Y = AX^b$ b = -0.21678

LN A = 8.1700 A = $e^{8.1700}$ = 3533.22

$Y_x = 3533.22\ X^{-.21678}$

If we want to know the slope:

b = -0.21678 slope = $2^b = 2^{-.21678}$ = .8605 = 86.05%

Cumulative Average Formulation

Definition: As the total quantity of units doubles, the cumulative average cost decreases by a constant percent

Formula: $\bar{Y}_x = AX^b$

where: $\bar{Y}_x$ **- Average cost of x number of units**

A - Cost of the first unit

X - Total number of units under consideration

b - slope coefficient b = ln(slope)/ln(2)

Production Breaks

Production Breaks

- **In analyzing production breaks two questions must be addressed**
  - **How much learning was lost**
    - **Anderlohr Method**
  - **How will this impact future lots**
    - **Retrograde method**

Anderlohr Method

George Anderlohr first published an article in Industrial Engineering in 1969 in which he divided all learning lost, by an organization, due to a break in production, into five categories

Anderlohr Categories

- **Personnel Learning**
  - **Physical loss, movement or layoff of personnel, loss of learning for people returning to a project**
- **Supervisory Learning**
  - **Same as above for supervisors, plus the familiarity with the previous workers**
- **Continuity of Production**
  - **Improvement in production line, work stations, location of bins, parts, etc**
- **Special Tooling**
  - **Tooling needing to be replaced due to wear, misplacement, breakage, and newer/better tooling**
- **Shop Methods/Instructions**
  - **Instructions that have been lost or need to be updated**

Weighting

- **Weights for each category should be developed in consultation with the contractor/experts.**
- **Example**

Personnel	30%
Supervisory	20%
Continuity of Production	20%
Special Tooling	15%
Methods	15%
TOTAL	100%

We will use these weightings to determine a Learning Lost Factor (LLF)

What is situation after break?

Contractor Provides the Following Info for the situation AFTER the break:

Personnel:	75% returned
	65% skill retained
Supervisory:	90% returned
	90% skill retained
Continuity of Production:	2 of 4 lines down
Special Tooling:	5% lost, worn or broken
Methods:	7% shop instructions need mod

How much is lost?

Contractor Provides the Following Info for the situation AFTER the break:

Personnel: 75% x 65% = 48.75% retained

So…1 – 48.75% = 51.25% LOST

Supervisory: 90% x 90% = 81% retained

So… 1 – 81% = 19% LOST

Continuity of Production:

2 of 4 lines torn down = 50% LOST

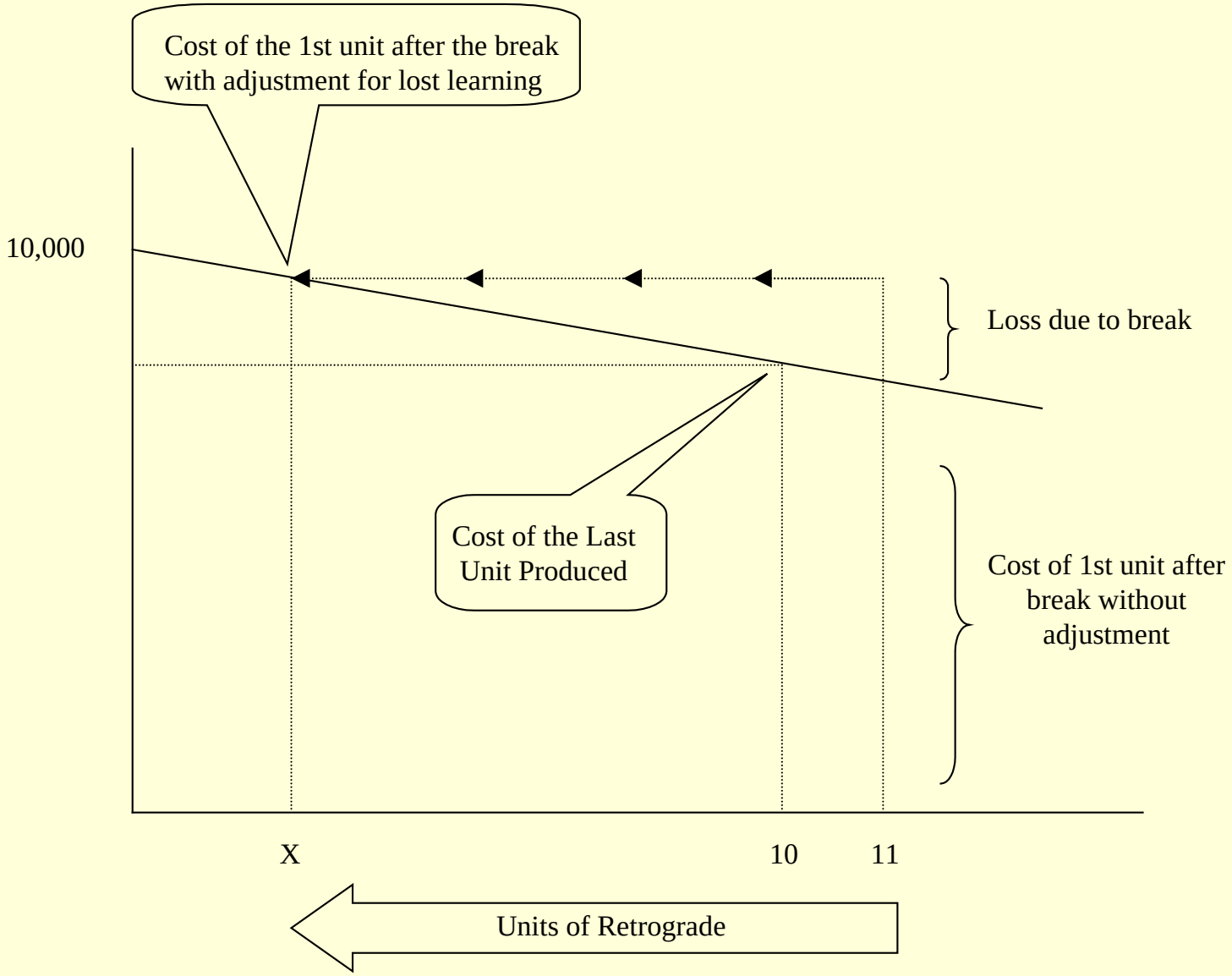
Special Tooling: 5% LOST

Methods: 7% LOST

Combine these to get LLF

Category	Weight	% Lost	Wtd Loss	
Personnel	0.30	0.51	0.1538	
Supervisory	0.20	0.19	0.0380	
Production	0.20	0.50	0.1000	
Tooling	0.15	0.05	0.0075	
Methods	0.15	0.07	0.0105	
TOTAL Lost Learning Factor (LLF)			0.3098	**31%**

How Retrograde Method Works



Retrograde Method Steps

- **Find the amount of learning achieved (LA)**

  $LA = Y_1 - Y_{\text{last unit before break}}$

- **Estimate the # hours of learning Lost**

  Learning Lost = LA * LLF

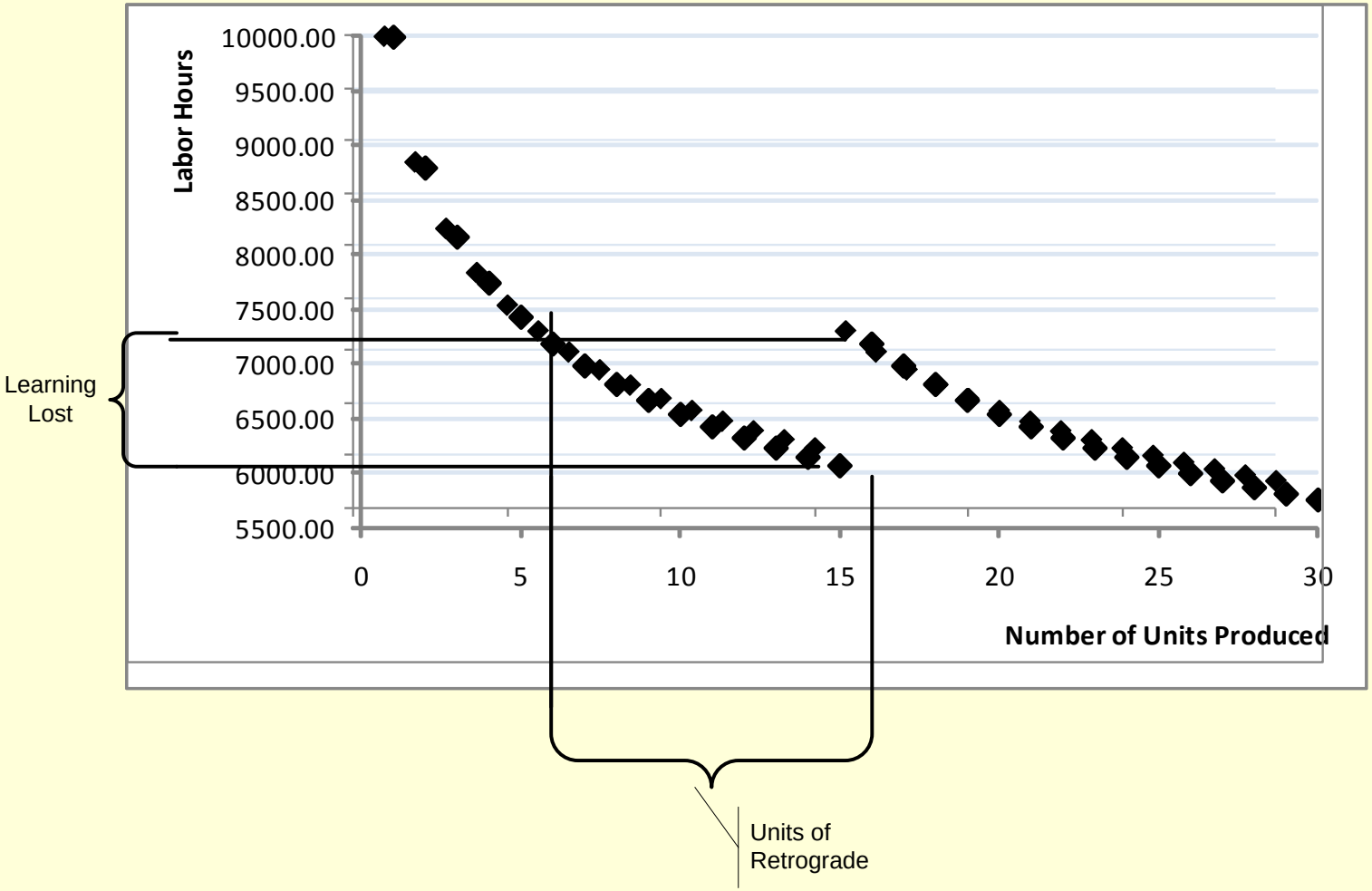
- **Estimate the new cost of the first unit after the break**

  Y' = Y + Learning Lost

- **Find the unit with the cost of Y'**
- **Determine the # of units of retrograde**
- **Estimate Lot Costs after the Break**

Scenario

Fifteen XH-99 helicopters were produced prior to a 6-month production break. T1 is 10,000 man-hrs; the slope is 88% Unit Curve. Estimate the cost of the next 10 units.



Summary

- Origin of Learning Curve Theory
- Areas/Conditions for Learning
- Unit & CUMAVE Formulations
- Production Breaks/Lost learning
- Other Areas to Explore
  - Production Rate Model
  - Fixed Cost Model

- Questions?

Backup

Total Lot Cost – Cum Average

If this $\bar{Y}_x = AX^b$ gives us the <u>average cost</u> how do we get the <u>total cost</u> under this theory for X number of units?

Total Cost = Average cost * # of units

$$CT_x = AX^b * X$$

$$CT_x = AX^{b+1}$$

Calculating the cost of a single Unit

Since we can calculate the total cost of X units, then we can solve for the cost of the Xth unit by taking the total cost of X units and subtracting the total cost of (X – 1) units

$$Y_x = AX^{b+1} - A(X-1)^{b+1}$$

$$Y_x = A[X^{b+1} - (X-1)^{b+1}]$$

Calculating Lot Costs – 1st Lot

If you are dealing with the first lot then just use the total cost equation

Total Cost = Average cost * # of units

$$CT_x = AX^b * X$$

$$CT_x = AX^{b+1}$$

Calculating Lot Costs – Not 1st Lot

If you are dealing with subsequent lots it gets a little more complicated

- **Steps**
  - **Calculate the total cost up to the last unit in the lot (L) using our equation**

$$CT_L = AL^{b+1}$$

  - **Calculate the total cost up all units up to F-1 using the same equation (we want to include the cost of F in the lot)**

$$CT_{F-1} = A(F - 1)^{b+1}$$

Calculating Lot Costs – Not 1st Lot Continued

- **Take the difference. What remains is the cost of units F through L.**

$$TC_{F,L} = AL^{b+1} - A(F-1)^{b+1}$$

- **Summarizing these steps in one equation:**

$$TC_{F,L} = A\left[L^{b+1} - (F-1)^{b+1}\right]$$