Student Guide

CON 170

Fundamentals of Cost and Price Analysis

Unit 2

Quantitative Methods for Contract Pricing

July 2013

Lesson Title	Quantitative Methods for Contract Pricing
Terminal Learning Objective	Given a contract pricing scenario, apply the appropriate quantitative method for contract pricing.
Time Required	8.5 hours
Method of Instruction	• Homework reading and research activity • Instructor-facilitated discussion • Classroom exercise
Lesson Schedule	Instructor-facilitated Discussion — 6.5 hours Classroom Exercise — 1.75 hours Lesson Summary/Summaries — 15 minutes
References, Supplemental Readings	Contract Pricing Reference Guide (CPRG), Volume 2
Evaluation Method	Student performance will be informally evaluated during class research activities and class exercises. Student performance will be evaluated on Exam 1.
Student Preparation Prior to Class	Read CPRG Volume 2, and complete Unit 2 Lesson 1.
Estimated Student Preparation Time	1 hour

Lesson Presentation

Introduction

In this unit, we will discuss mathematical terminology, symbols and definitions, and their application to cost and price analysis.

Terminal Learning Objective

2.1 Apply the math, algebra and statistics methods from Unit 2 Lesson 1 in quantitative pricing methods for contract pricing

Enabling Learning Objectives

At the conclusion of this unit, you will be able to:

- Lesson 1 -- Compute fundamental quantitative problems
- Lesson 2 -- Price Indexing
  ELO 2.01 Use Price Indexing to make price adjustments necessary to analyze price and cost information collected over a period of time.
- Lesson 3 -- Analyzing Data Shape, Center, Spread and Trend
  ELO 2.02 Given a set of data, Analyze Data Shape, Center, Spread and Trend
  characteristics.
- Lesson 4 -- Net Present Value
  ELO 2.03 Given a set of data, calculate the Net Present Value of the given data.
- Lesson 5 -- Cost Estimating Relationships
  ELO 2.04 Given Market Research data, calculate and identify reasonable Cost
  Estimating Relationships.
- Lesson 6 -- Cost Volume Analysis
  ELO 2.05 Through Cost-Volume Analysis, recognize the nature of fixed, variable, semi-variable and total costs, and develop a price estimate.
  ELO 2.06 Through Cost-Volume Analysis, determine a proposed price to be rational or irrational with respect to a “buy-in” seller strategy.
- Lesson 7 -- Cost Volume Profit Analysis
  ELO 2.07 Through Cost-Volume-Profit analysis, recognize the nature of profit, revenue, contribution income, and calculate the contractor’s “break even” point

Lesson 1 – Compute fundamental quantitative problems

1. Given the following equations, solve for X:

1a. If a = 6, b = 15, c = 5, solve for X: a(X) = b + c(X)

1b. If a = 20, b = 70, c = 10, solve for X: a(X) = b + c(X)

2. For the equations below, answer the following, and use the graph on the next page:

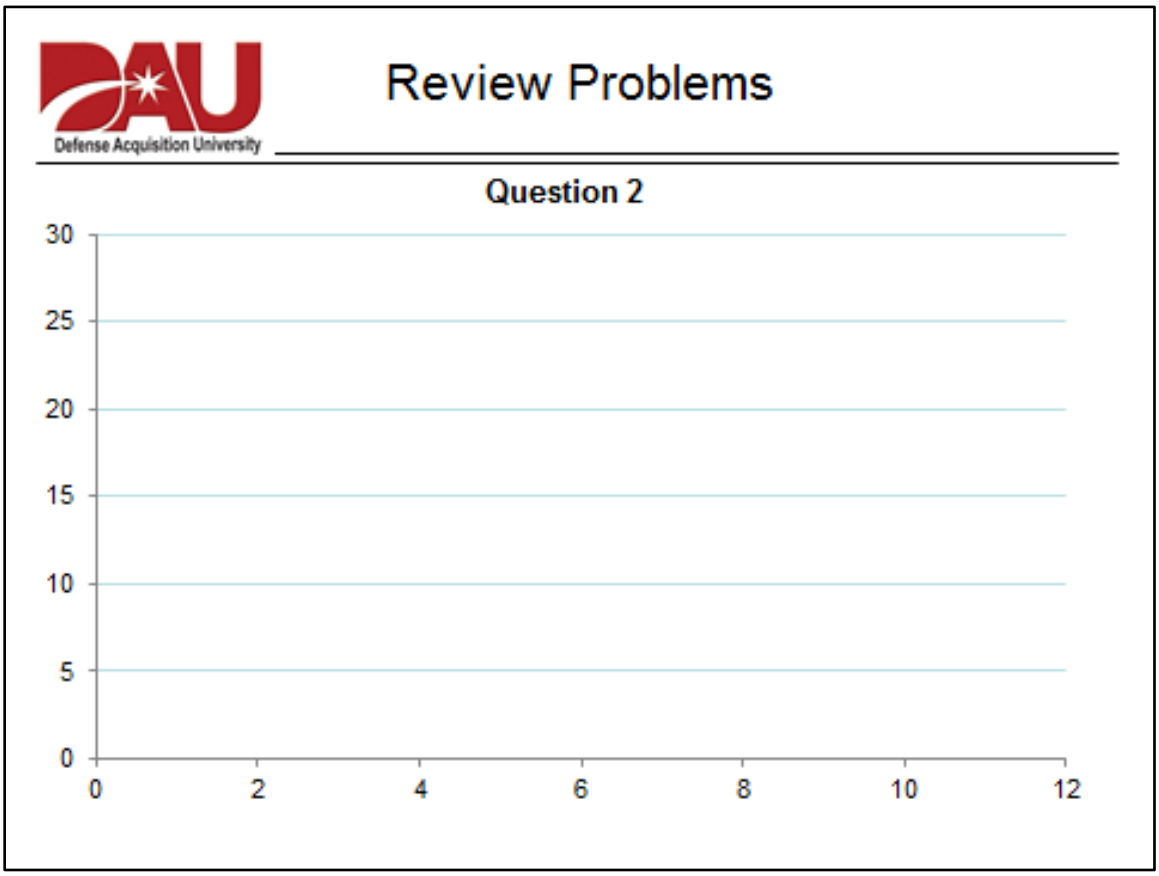
2a. Graph the following, Y = 4 + 2(X), where

X	Y
1	?
3	?
5	?
7	?
9	?
11	?

2b. On the same graph as 2a, graph the following:
Y = 10 + 1(X), where

X	Y
0	?
2	?
4	?
6	?
8	?
10	?

2c. Look at the graph. At what point do the lines appear to intersect?



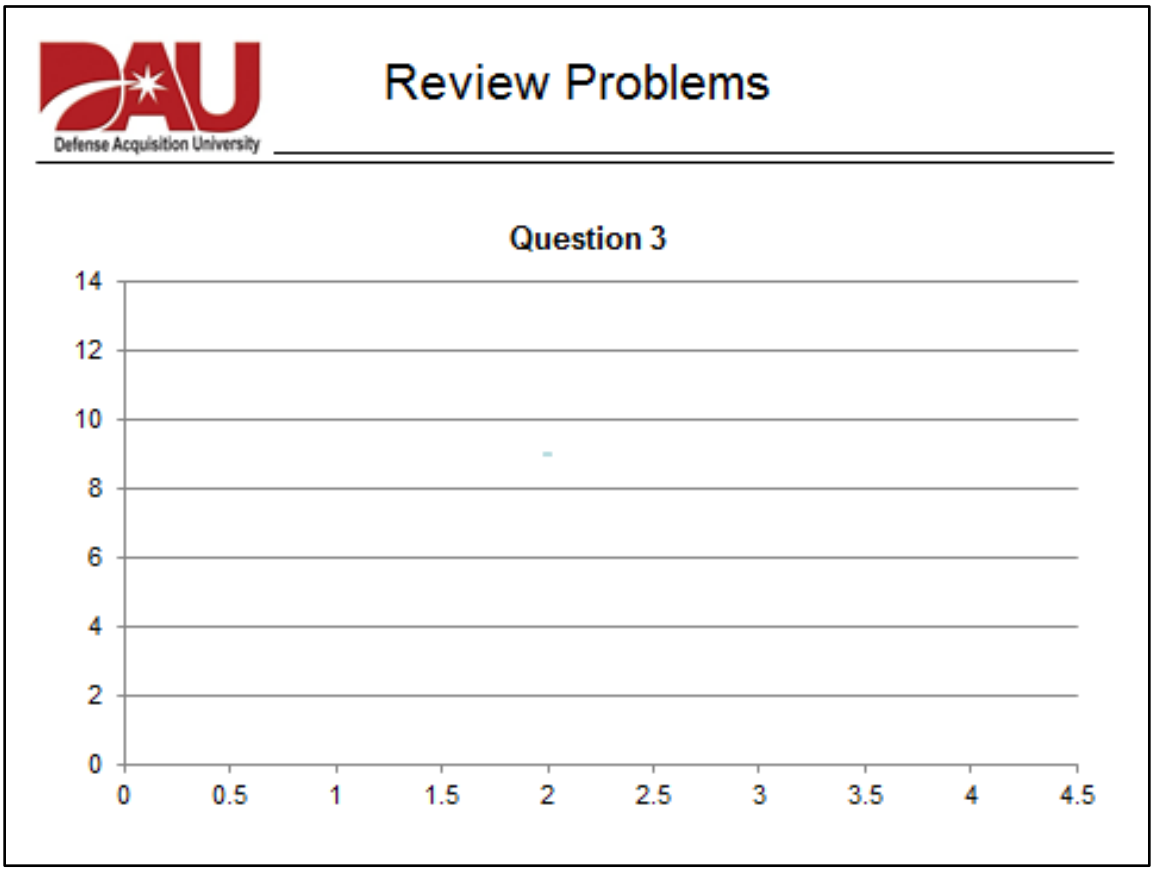
2d. Find the value for X which makes the two equations equal

$$4 + 2(X) = 10 + 1(X)$$

2e. How do your responses to 2c and 2d compare?

3. Given Y = A + BX, and A = 5, and B = 2, calculate Y at the following X coordinates. Note: an equation in this form represents a line, and can be called a "linear equation." Graph the following points:

X	Y
0	?
1	?
2	?
4	?



With respect to the line, what does B represent? *Hint: slope is "rise over run," or, the change in y for every change in X. It can be calculated as follows*:

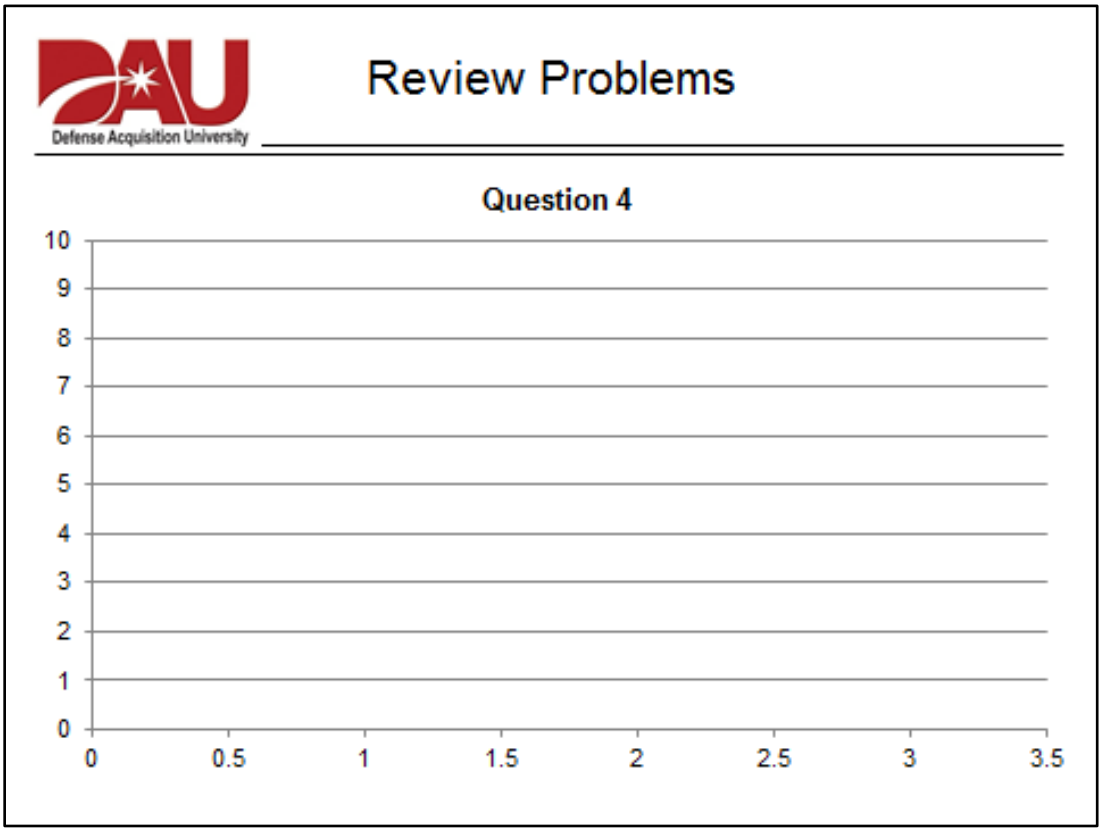
$$\frac{Y_2 - Y_1}{X_2 - X_1}$$

What does A represent? *Hint: the y-intercept is the value of Y when X = 0.*

Write the equation of this line.

4. Given Y = A + BX, where A = 3 and B = 2, calculate the coordinates and plot the graph for:

X	Y
0	?
1	?
2	?
3	?



Write the equation of this line.

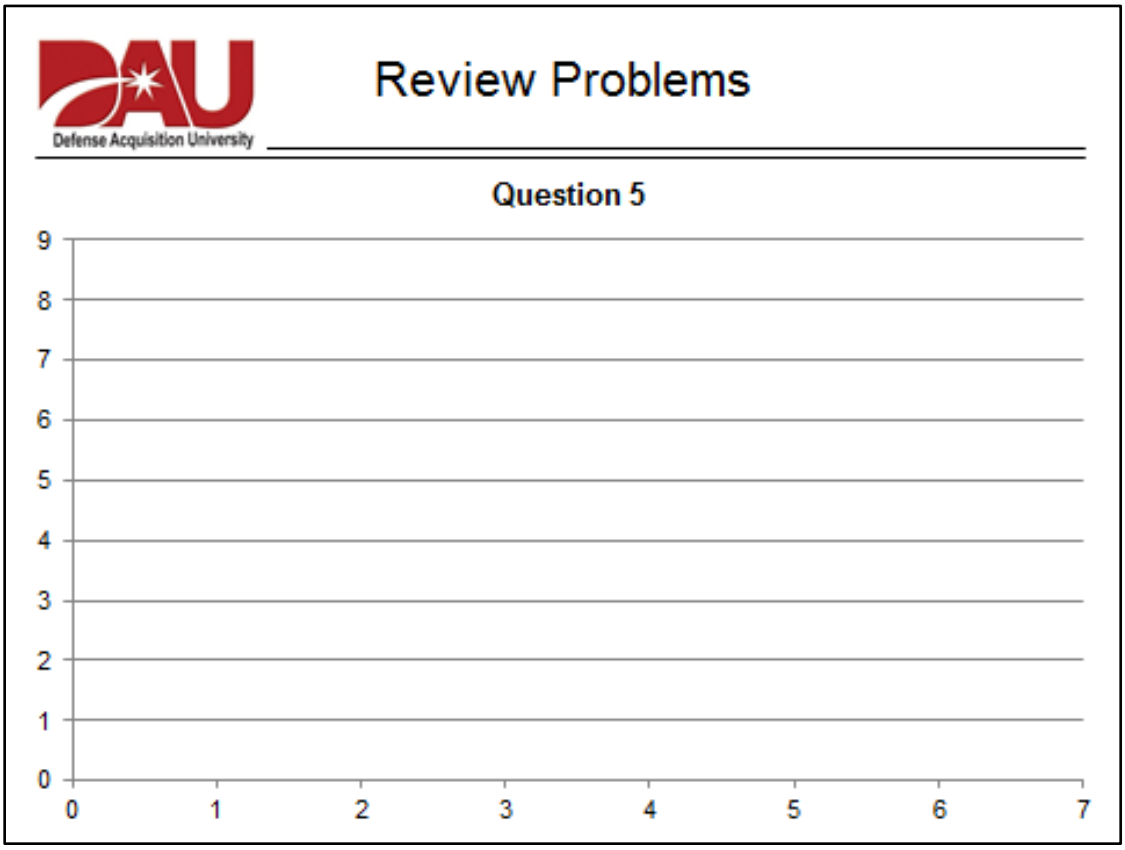
What is the "y-intercept"? *Hint: the y-intercept is the value of Y when X = 0.*

What is the slope of this line? *Hint: slope is "rise over run" or, the change in y for every change in X. It can be calculated as follows:*

$\frac{Y_2 - Y_1}{X_2 - X_1}$

5. Given the following points, plot the graph.

X	Y
1	3
2	4
3	5
6	8



5a. What is the slope of this line?

Hint: $\frac{Y_2 - Y_1}{X_2 - X_1}$

5b. What is the y-intercept? *Hint: the y-intercept is the value of Y when X = 0.*

5c. With the slope and the y-intercept, write the equation of this line:

5d. With the equation of the line, verify the points you plotted earlier fit the line of your graph.

$Y = 2 + 1X$

X	Y
0	
1	
2	
3	
6	

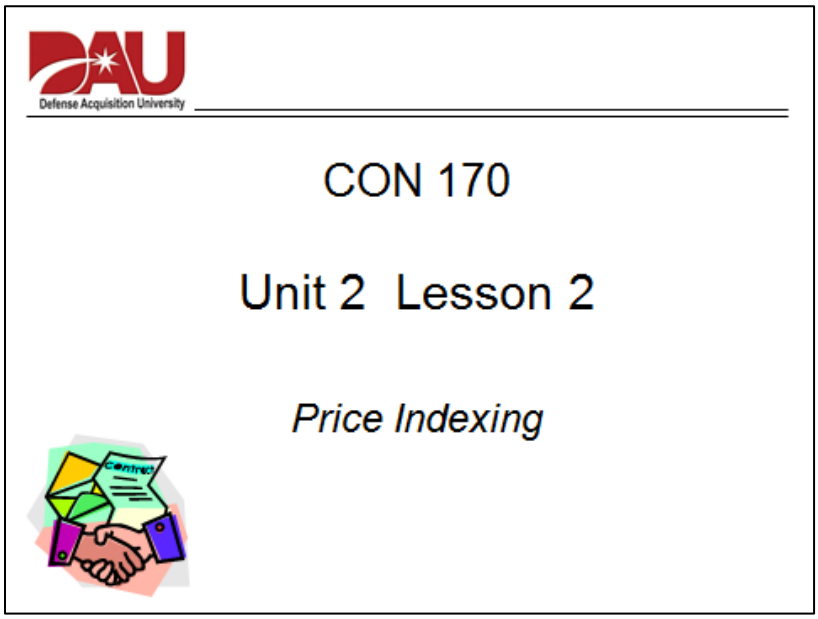
6. Lester must earn at least a 90% average in his agency's Contracting Officer warrant board course in order to earn his Procuring Contracting Officer (PCO Warrant). With all exams weighted equally, based on his scores below, will Lester earn his PCO warrant?

Exam 1	84%
Exam 2	86%
Final Exam	96%

7. Lester wakes up after a bad dream, and realizes he misread the PCO course syllabus. While he must earn at least a 90% in his Contracting Officer warrant board course to earn his Procuring Contracting Officer (PCO Warrant), his first two tests are worth 25% of his grade, and the final exam is worth 50%. Based on his scores, will Lester earn his PCO warrant?

Exam 1	84%
Exam 2	86%
Final Exam	96%

Lesson 2 -- ELO 2.01 Use Price Indexing to make price adjustments necessary to analyze price and cost information collected over a period of time.



Introduction

Price Analysis involves the comparison of the apparent successful quote/proposal to other prices/costs. Sometimes we have to compare the quote/offer to historical prices/costs. The changing value of the dollar over time can complicate comparisons and other analysis using price or cost information collected over time. You can use price indexes to adjust prices/costs to compensate for inflation or deflation and facilitate comparisons and other analysis.

Price Index Numbers Defined

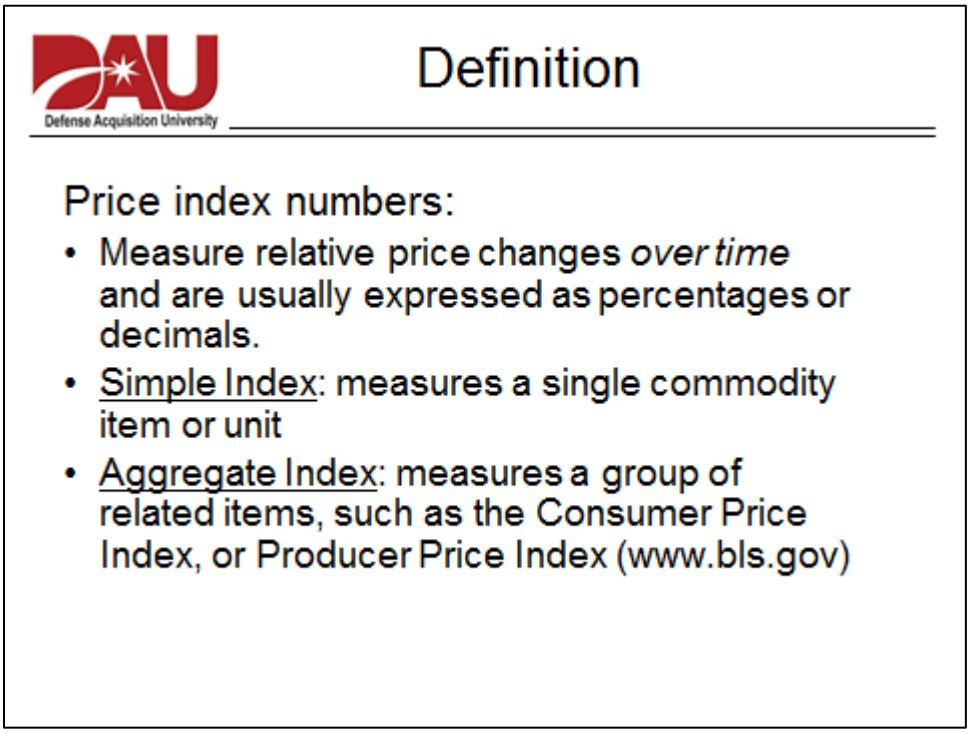
Probably the most commonly used quantitative measure used in contract pricing is the price index number. **Price index numbers measure relative changes in prices over time**. Index numbers are essentially a ratio of the price of goods today compared to their price in previous years.

While index numbers can measure both prices and costs, discussions of index numbers in contract pricing normally refer to price indexes. In addition, indexing is commonly used in pricing to measure productivity.

Simple and Aggregate Price Index Numbers - Price index numbers can indicate price changes for one or several related items or services over a period of time.

> **Simple index numbers calculate price changes for a single item over time.** Index numbers are more accurate if they are constructed using actual prices paid for a single commodity, product or service rather than the more general aggregated index.

- **Aggregate index numbers calculate price changes for a group of related items over time.** Aggregated indexes permit analysis of price changes for the group of related products, such as price changes for apples, oranges, plywood, or nails. An example of an aggregate price index is the *Producer Price Index* (Bureau of Labor Statistics) that provides information on the changes in the wholesale price of products sold in the United States over a given period of time.



Identifying Situations for Use

Inflate/deflate prices or costs for direct comparison. You can use price index numbers to estimate/analyze a product's cost or price today using the cost or price of the same or a similar product in the past.

Inflate/deflate prices or costs to facilitate trend analysis. You can use index numbers to facilitate trend or time series analysis of costs or prices by eliminating or reducing the effects of inflation so that the analysis can be made in constant-year dollars (dollars free of charges related to inflation/deflation).

Estimate/project prices or costs over the period of contract performance. Prices and costs of future performance are not certain. One effect that you must consider is the changing value of the dollar. You can use index numbers to estimate and negotiate future costs and prices.

Adjust contract price or cost to account for inflation, deflation, and other market factors. When price changes are particularly volatile, you may need to include an economic price adjustment clause in the contract. The use of index numbers is one of the most popular methods used to identify and define price changes for economic price adjustment.

Situations for Use

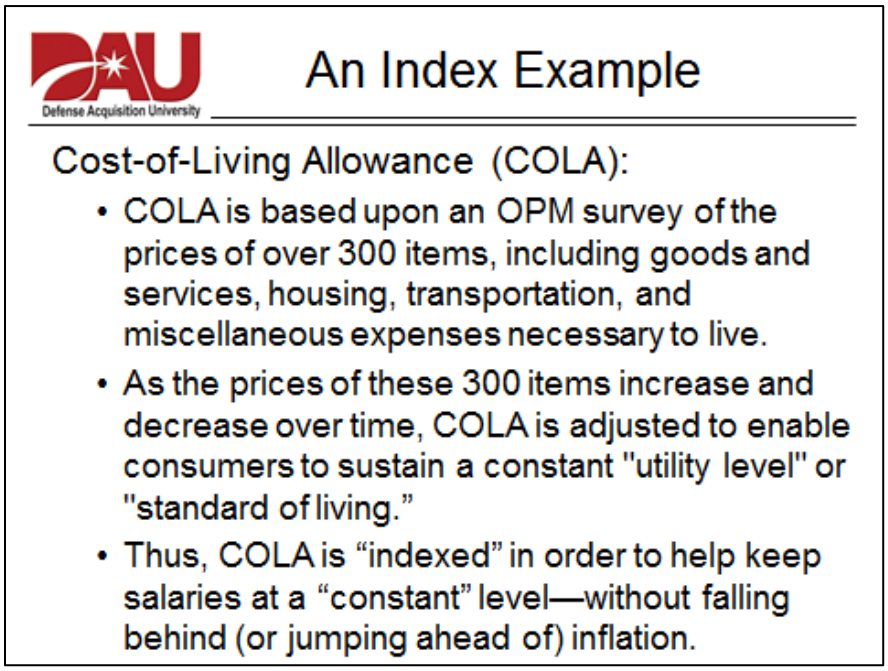
Use price index numbers to:

- Inflate/deflate prices for direct comparison
- Inflate/deflate costs to facilitate trend analysis
- Estimate/project prices or costs over the contract period
- Adjust contract price or cost for inflation, deflation and other market factors

Adjusting Historical Prices to "Current Year Dollars"

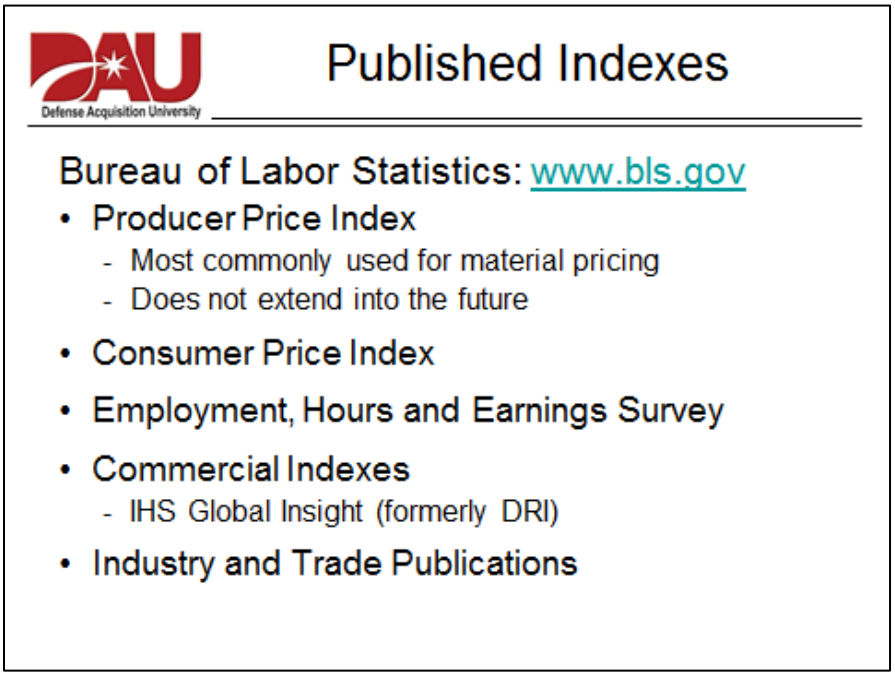
- Use price index numbers to adjust historical prices to estimate what the price would be in more recent or current years.
- This process (called "indexing") adjusts price estimates to account for the effect of inflation and other factors
- Enables buyers to compare current and historical prices in "current year" dollars.

An example of a common index is the Cost of Living Allowance for many Government and contractor employees.



Selecting a Price Index for Analysis

There are many sources of published indexes. You may not have the time or data required to construct the price indexes that you need for price or cost analysis. Fortunately, there are many sources of previously constructed price indexes that you can use to estimate price changes. These sources include:



Points to Consider in Index Selection

Use published indexes carefully, since a published index will usually not exactly fit the pattern of price changes for the product or service that you are analyzing. The data isn't usually from a specific contractor or location, but represent national or regional averages. Nevertheless, pre-constructed index numbers offer a practical alternative to the costly and time-consuming task of developing index numbers from basic cost data.

When you use published indexes, choose the index series that best fits your specific analysis effort. Usually, the closer the chosen index series relates to the item that you are pricing, the more useful the number will be in your analysis.

If you are buying a finished good, indexes representing raw materials and purchased components will not necessarily provide an accurate basis for projecting prices because the finished good may also be strongly influenced by trends in direct labor, cost of capital, etc. Accuracy can be improved through the use of a weighted average index, which represents changes in both labor and material elements of price. Many contracting organizations develop weighted average indexes for major products or major groups of products.

Indexes from the Bureau of Labor Statistics. The Government collects and publishes vast amounts of data on prices. The U.S. Department of Labor, Bureau of Labor Statistics (BLS) is the principal fact-finding agency for the Federal Government in the broad field of labor economics and statistics. (See http://www.bls.gov/) Four of the best known sources of index numbers are published by the BLS:

> **Producer Price Index.** Probably the best known and most frequently used source of price index numbers for material pricing is the *Producer Price Indexes (PPI)* published monthly by the BLS. These indexes report monthly price changes at the producer/wholesale level for 15 major commodity groups, and over 30 services. The table below includes several of the commodities and services addressed by the PPI, and can be found at http://bls.gov/web/ppi/ppitable06.pdf. A producer price index for an industry is a measure of changes in prices received for the industry's output sold outside the industry (that is, its net output). Producer price indexes measure the average change in prices received by domestic producers of commodities in all stages of processing. The PPI is an output price index, that is, it measures price changes received by manufacturers of a product. It is neither a buyer's index nor an input price index, that is, it does not measure the cost of producing that item. PPI data are based on selling prices reported by establishments of all sizes selected by probability sampling, with the probability of selection proportionate to size. Individual items and transaction terms from these firms are also chosen by probability proportionate to size sampling methods. PPIs are based on a monthly sample of about 100,000 quotations, resulting in publication of over 10,000 different indexes each month (for more information, see the BLS' Producer Price Index, Escalation Guide for Contracting Parties, on the BLS website).

PRODUCER PRICE INDEXES COMMODITY GROUPS	
Commodity Code	**Commodity or Service Description**
01	Farm Products
03	Textile Products and Apparel
04	Hides, Skins, Leather, and Related Products
05	Fuels and Related Products and Power
06	Chemicals and Allied Products
07	Rubber and Plastic Products
08	Lumber and Wood Products
09	Pulp, Paper, and Allied Products
10	Metals and Metal Products
11	Machinery and Equipment
12	Furniture and Household Durables
13	Nonmetallic Mineral Products
14	Transportation Equipment
30	Transportation Services
32	Warehousing Storage Services
38	Data Processing and Related Services
45	Professional Services (partial, including business, legal, engineering, IT, logistics consulting)
80	Construction (partial)

For a more specific look at information available under the Producer Price Index, execute the following steps from the www.bls.gov website:

Defense Acquisition University

Example: Producer Price Index
for Ball and Roller Bearings

1. Go to www.bls.gov, upper left, click on "Subject Areas"
2. Upper left, under "Inflation & Prices," click on "Producer Price Indexes"
3. Upper left, under "Browse PPI," click on "PPI Databases"
4. Under "PPI Databases" table, in the "Industry Data" row, click on "Top Picks"
5. Scroll down to "Ball and Roller Bearing Manufacturing," and click in the box
6. Then, scroll down and click on "Retrieve Data"
7. "Index Numbers" in the table are used to adjust historical prices to estimate current prices in this industrial area.

At Step 7, you will find actual tables of index numbers recorded by the Bureau of Labor Statistics, unique to the category of ball and roller bearing manufacturing. In addition to this Ball and Roller Bearing category, you can explore many other categories within a short time.

Consumer Price Index Detailed Report. The consumer price index (CPI), published monthly in the *Consumer Price Index Detailed Report*, reports on changes in consumer prices for a fixed mix of goods selected from the following categories:

- food
- clothing
- shelter and fuels
- transportation
- medical services

You should normally **not use the CPI to adjust material prices** because the CPI reflects retail rather than wholesale price changes. However, the CPI can be of value in pricing services when labor rate increases are linked to changes in the CPI.

Monthly Labor Review. *The Monthly Labor Review* is edited and posted by the Bureau of Labor Statistics, and includes analysis of the labor costs, output, productivity, unemployment for different skills and population demographics. The Review also includes selected data from a number of Government indexes, including:

- Employment Cost Index
- Consumer Price Index
- Producer Price Indexes
- Export Price Indexes
- Import Price Indexes

That data and other information presented in the publication can prove useful in conducting market research and for analyzing the price of contracts, such as service contracts, where direct labor is a significant part of contract price.

Employment, Hours, and Earnings Survey. The Employment, Hours, and Earnings Survey presents information on the hours worked and an earnings index for various classes of labor. Like the *Monthly Labor Review*, the survey can be very useful in market research and for pricing contracts in which direct labor is a significant part of the contract price.

Indexes from Other Government Agencies. Data on contract prices are also available from agencies other than the Bureau of Labor Statistics. The most notable are the Federal Reserve System and the Bureau of Economic Analysis.

Federal Reserve System. The Board of Governors publishes the *Federal Reserve Bulletin*, which includes economic indexes and data on business, commodity prices, construction, labor, manufactures, and wholesale trade. Each bank in the system publishes information

each month with special reference to its own Federal Reserve District. (See http://www.federalreserve.gov/pubs/bulletin/default.htm)

Bureau of Economic Analysis Publications. The Bureau of Economic Analysis, Department of Commerce, publishes the *Survey of Current Business* and the *Business Conditions Digest*. The *Survey of Current Business* provides general information on trends in industry and the business outlook. It furnishes economic indexes on business, construction, manufactures, and wholesale trade. The *Business Conditions Digest* presents almost 500 economic indicators in a form convenient for analysis, as well as different approaches to the study of current business conditions and business prospects, including leading economic indicators. (See http://www.bea.doc.gov/)

Indexes from Government Contracting Organizations. Many Government contracting organizations have teams of analysts who develop indexes that are particularly applicable to the organizations' specific contracting situations. These indexes may be developed from raw price data, or they may be developed as weighted averages of published indexes.

Indexes from Commercial Forecasting Firms. Numerous commercial indexes are available for use in contract price analysis. While most Government indexes only report historical price changes, many commercial indexes also forecast future price movement. In situations where forecasts are necessary, commercial indexes may prove particularly useful. A major source for this information is Global Insight (formerly DRI). Before using such indexes, examine their development and consult with auditors, technical personnel, and other contracting professionals to assure that they are applicable in your analysis situation.

Indexes from Industry. Industry and trade publications frequently provide general forecasts of economic conditions and price changes anticipated in the industry. To identify which publications have economic information relevant to a particular product, ask Government technical personnel. Companies can also assist you in the identification of appropriate publications. However, be sure to verify with Government personnel the appropriateness of sources of information recommended by responders.

Indexes from Newspapers. Publications, such as local, national, and financial newspapers, provide valuable forecasts of price changes in specific industries. The information reported is normally data provided by the Government, economic forecasting firms, or industry groups.

Adjusting Prices for Analysis

There are two calculations we are interested in relative to indexing. First, we will learn to calculate the relative price change between two periods. Next, we will use index numbers to estimate current prices. Making these calculations requires the use of the following symbols.

NI:	New Index	for the time we are calculating an estimate
OI:	Old Index	from the time of the historical data element
NP:	New Price	what we are trying to calculate an estimate for
OP:	Old Price	the historical price paid

First, in calculating the relative price change of over a period of time, we may use index numbers, or historical prices from our market research.

Percentage Change Between Two Periods

Use indexing to calculate the *percentage change in prices*:

- You have index numbers for a certain commodity from 2008 through 2013. Your boss asks you to estimate the percentage change in the price in the commodity from 2008 to 2013. Use the following formula:

$$\left[\frac{\text{NI in 2013}}{\text{OI from 2008}} \times 100 \right] - 100 = \text{the Percentage Change}$$

- Or, you have historical prices of a certain commodity from 2008 through 2012. Your boss asks you to estimate the percentage change in the price of the commodity from 2008 to 2012. Use a similar formula:

$$\left[\frac{\text{New Price in 2013}}{\text{Old Price from 2008}} \times 100 \right] - 100 = \text{the Percentage Change}$$

Calculate the relative price change (or, "percentage change") between two periods. Index numbers indicate the percentage change in prices relative to the base year. Using the table of index numbers below, we can calculate the percentage change in a product's price between certain years.

YEAR	PRODUCT INDEX
2008	100.0
2009	105.3
2010	112.0
2011	116.5
2012	119.3
2013	123.2

To adjust prices for inflation or deflation, you must be able to do more than determine how prices have changed relative to a base year. You must also be able to determine how prices changed between any two time periods.

$$[\ (NI\ /\ OI) \times 100\ ] - 100 = \text{Percentage Change}$$

Where:
NI = New Index
OI = Old Index
(*Chronological Indexes)*

Example 1: Looking at the table above, how did prices change between 2010 and 2013? To calculate the percentage price change between any two time periods, you must follow the same procedure that you would follow if you had actual price data; you must divide.

[(NI / OI) × 100] – 100 = Percent Change

[(123.2 / 112.0) × 100] – 100 = 10% increase

Based on the price index and this calculation, a buyer could estimate that product prices in 2013 were 1.10 times the prices in 2010, or 10.0 percent more than the prices in 2010. The product index numbers enable a buyer to estimate a general price increase in this product of about 10% due to inflation as well as other factors unique to building and selling this product.

Example 2: You are conducting market research to purchase high-powered microscopes for your laboratory. In 2010, the microscope price was 15,000. In 2013, the price for the same microscope is now 16,500. What is the percentage change since 2010? Notice, in this example, we do not have index numbers, but we have historical prices. Therefore, we calculate as follows:

[(NP / OP) × 100] - 100 = Percentage Change

Where:
NP = New Price
OP = Old Price
(*Chronological Indexes)*

[($16,500 / $15,000) × 100] - 100 =

[1.1 × 100] - 100 =

= 10% increase

<u>Try it!</u> Using index numbers in the table above, calculate the percentage change based on the indexes between 2009 and 2011.

[(NI / OI) × 100] - 100 = ?

Second, we can estimate prices in a current or future period based on prices from a previous period using index numbers. We also use index numbers to estimate prices in current years. When we obtain historical prices through market research, we can use index numbers estimate the price of the item in the current year.

The price adjustment formula, used to estimate current or future prices from historical data is as follows:

$$(NI \ / \ OI) \ x \ OP \ = \ NP$$

Where:
NI = New Index, representing the more current year
OI = Old Index, representing the year of the historical price
OP = Old Price, which is the historical price paid
NP = New Price, which is the price estimate for the more current year

Example 3: Given the table of index numbers, what would you estimate the price of a product to be in 2013 if the price in 2010 was $1,000? Note: this is similar, but <u>not the same</u> as the <u>percentage change</u> calculation! To estimate a <u>price</u>, use the price adjustment formula:

$$(NI \ / \ OI) \ x \ OP \ = \ NP$$

$$(123.2 \ / \ 112) \ x \ \$1{,}000 \ = \ \$ \ ?$$

$$(123.2 \ / \ 112) \ x \ \$1{,}000 \ = \ \$1{,}100$$

YEAR	PRODUCT INDEX
2008	100.0
2009	105.3
2010	112.0
2011	116.5
2012	119.3
2013	123.2

Based on the price index numbers and a historical price, this calculation indicates a buyer would expect to pay approximately $1,100 for this product in 2013. This increase in price from 2010 can be attributed to inflation as well as other factors unique to building and selling this product.

<u>Try it!</u> Estimate the price for a product in 2011 based on a historical price of $12,000 paid in 2009, using the index numbers in the table from the previous example:

$$(NI \ / \ OI) \ x \ OP \ = \ NP$$

<u>Try Another One</u>. Using the same table of index numbers above, estimate the price of the product in 2013, based on the historical price of $10,400 in 2010.

$$(NI \ / \ OI) \ x \ OP \ = \ NP$$

Now that you have calculated the price estimate for the product in 2013, consider the offeror's proposed price of $11,083. As the Contracting Officer, would you consider this price to be fair and reasonable? Defend your answer.

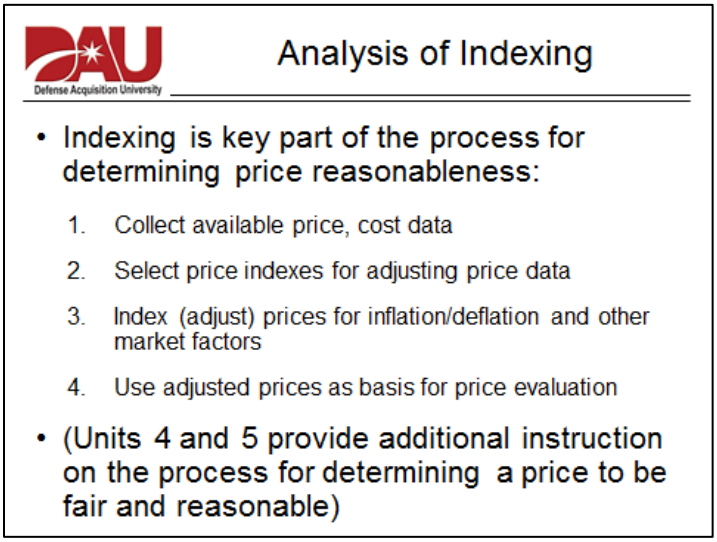
Discussion. This example reveals that making the determination on price reasonableness must not be based on indexing analysis alone. We must consider the entire contracting situation, including:

- Recent changes in market conditions, including area of technology, levels of competition, overall demand for a product
- Differences in order quantity, delivery and packaging requirements;
- Geographic location of manufacturing, delivery
- Quality, performance parameters, warranty
- Other contract terms, such as warranty, labor agreements, acquisition strategy
- Age of the historical data from market research.

Adjusting for such differences will be covered in more detail in CON 270.

Class Exercises. Now that we have covered the key steps in using price indexing, review Exercise 1 using the following 4-step process:

Step 1. Collect available price data.
Step 2. Select price indexes for adjusting price data.
Step 3. Adjust prices/costs for inflation/deflation and other market factors.
Step 4. Use adjusted prices for pricing comparisons.



Exercise 1. Using Price Indexing to Analyze Price-Reasonableness. A buyer is analyzing a contractor's proposed price of $23,000 for a turret lathe to be delivered in 2013.

Step 1. Collect Available Price/Cost Data. Market research reveals a historical purchase of the same machine tool in 2010 at a price of $18,500. Determine whether the 2013 proposed price of $23,000 is reasonable.

Step 2. Select an Index Series For Adjusting Price/Cost Data. Select or construct an appropriate index series. During market research, search for the common indexes used by Government and industry when conducting business in this area. In this case, you might search the Bureau of Labor Statistics, or another commercial index which relates to metal working machinery or manufacturing, under the general NAICS code of 3335. In this example, we have established a Machinery and Equipment Index in the table below to help us assess price movement for turret lathes over the past several years.

YEAR	MACHINERY AND EQUIPMENT INDEX
2008	100.0
2009	103.3
2010	106.0
2011	110.8
2012	115.0
2013	121.9

Step 3. Adjust Prices/Costs for Inflation/Deflation. After you have selected an index, you can adjust prices to a common dollar value level. In this case, you would normally adjust the historical 2010 price to the 2013 dollar value level. To make the adjustment, you simply use one of the methods already demonstrated.

Using the Formula Adjustment Method.

(NI / OI) x OP = NP

(121.9 / 106.0) x $18,500 = $21,275

By using index numbers and a historical price, a buyer can estimate the new price in 2013 to be $21,275. This new price accounts for inflation and other factors unique to manufacturing this item.

Step 4. Make Direct Price Comparison. Once you have made the adjustment for inflation/deflation, you can compare the offered and historical prices in constant dollars. The offered price/cost is $23,000, but the adjusted historical price/cost is only $21,275. Thus, the offered price/cost is $1,725, or 8.1 percent higher than what you would expect, given the historical data and available price indexes. What would you do next?

Remember: in addition to indexing, remember to also consider factors unique to each contracting situation.

Your objective in reviewing these unique areas is to understand significant variance between the indexed price estimate and the proposed price, make the determination of fair and reasonable price, and decide whether to enter negotiations or make award.

Exercise 2. In 2013, you are evaluating an offer of $22,500 each for five precision presses. Through market research, you discovered the "Machinery and Equipment Index" (Column B below) used by Government and industry analysts when purchasing these presses. In addition, your customer provided several historical prices paid, all the way back to 2008 (Column D below). Each of these purchases was for five presses, with similar performance and contract terms. Given this information, use price indexing to analyze if the proposed price of $22,500 appears to be reasonable.

A	B	C	D	E
Year	**Machinery and Equipment Index**	**Index Adjustment Calculation (NI / OI)**	**Historical Prices OP**	**Price Adjusted to 2013 NP**
2008	100.0		$17,391	
2009	103.3		$17,796	
2010	106.0		$18,087	
2011	110.8		$18,724	
2012	115.0		$19,245	
2013	121.9	--	--	?

Step 1. Collect Available Price/Cost Data. The organization has purchased five similar presses each year since 2008. The historical unit prices are shown in Column D of the table below. While purchase quantity changes are not present in this situation, unit prices are used to limit the effect of quantity differences on trend analysis. In this case, the only apparent cost/price trend in the unadjusted data is increasing prices.

Step 2. Select Price Indexes for Adjusting Price/Cost Data. In this example, we will use the same Machinery and Equipment Index we used in Example 1.

YEAR	MACHINERY AND EQUIPMENT INDEX
2008	100.0
2009	103.3
2010	106.0
2011	110.8
2012	115.0
2013	121.9

Step 3. Adjust Prices for inflation/deflation and other market factors. Using Columns C and D, calculate the adjusted price for the precision press in 2013 dollars in Column E.

Step 4. Make Price Comparisons. What trends do you observe?

What concerns will you address with additional market research?

In Closing, Remember the Issues and Limitations with Indexing. As you perform price/cost analysis, consider the issues and concerns identified in this section, whenever your analysis is based on data collected over time.

Were prices/costs collected over time adjusted for inflation/deflation? Inflation/deflation can mask underlying price changes. Price indexes should be used to compensate for the effect these general price changes.

Is it reasonable to use the price index series selected? The price index series selected for making the price/cost adjustment should be as closely related to the item being considered as possible. For example, you should not use the Consumer Price Index to adjust for changes in the price of complex industrial electronic equipment.

Are adjustments calculated correctly? Anyone can make a mistake in calculation. Assure that all adjustments are made correctly. This is particularly important when the adjustment is part of a contractor's offer or part of an analysis performed by other Government personnel.

Is the time period for the adjustment reasonable? When adjusting historical prices for inflation, take care in selecting the period of adjustment. There are two basic methods that are used in adjusting costs/prices, period between acquisition dates and the period between delivery dates. The period between acquisition dates is most commonly used because purchase dates are typically more readily available. However, be careful if delivery schedules are substantially different.

Is more than one adjustment made for the same inflation/deflation? For example, it is common for offerors to adjust supplier quotes to consider inflation/between the time when the quote was obtained and the date that the product will be required. This is acceptable unless the supplier already considered the inflation/deflation in making the quote.

How far into the future should you forecast? The further into the future that you forecast, the greater the risk.

Did we consider adjusting our price estimate based on unique factors, which may not be captured in an index, such as:

- Recent changes in market conditions, including area of technology, levels of competition, overall demand for a product
- Differences in order quantity, delivery and packaging requirements;
- Geographic location of manufacturing, delivery
- Quality, performance parameters, warranty
- Other contract terms, such as warranty, labor agreements, acquisition strategy
- Age of the historical data from market research.

Price Indexing Practice Problems:

You have been asked to analyze changes in prices and costs over the past several years, based on the Producer Price Index for a certain commodity category. Use the following Widget Producer Price Index factors to answer the questions below:

Year	Widget PPI
2008	100.0
2009	105.0
2010	98.0
2011	106.5
2012	112.9
2013	122.0

1. By what percentage did the indexes change between 2009 and 2013?

2. If an item was priced at $5,000 in 2009, estimate the price of the item in 2013.

3. An item was priced at $3,000 per unit in 2009. What would you estimate that the price was in 2010?

4. It is 2013 and you have just received a proposal from Brown Industries for 1,000 widgets at a price of $850 each. You remember that once before you purchased widgets. You pull the contract file and see that the price of $650 each for 1,500 widgets was determined fair and reasonable in 2011. Based on the Producer Price Index, what would your estimate be for the price in 2013?

Therefore, does the proposed price appear to be fair and reasonable? Defend your determination.

Unit 2 Lesson 2 Practice Problem Answers:

1. By what percentage did the indexes change between 2009 and 2013?
[(122 / 105) × 100] - 100 = 16.2%

2. If an item was priced at $5,000 in 2009, estimate the price of the item in 2013.
122 / 105 × $5,000 = $5,809.50

3. An item was priced at $3,000 per unit in 2009. What would you estimate that the price was in 2010?
[(98 / 105) × $3,000] = $2,800 Note, in this case, the price actually drops. It is possible that overall inflation was still positive, but other factors unique to the widget market (such as decreased demand or more competition) still caused the prevailing market prices to fall. Help students realize indexing with a specific index can reflect more about prices than simply inflation or deflation.

4. It is 2013 and you have just received a proposal from Brown Industries for 1,000 widgets at a price of $850 each. You remember that once before you purchased widgets. You pull the contract file and see that the price of $650 each for 1,500 widgets was determined fair and reasonable in 2011. Based on the Producer Price Index, what would your estimate be for the price in 2013?
(122 / 106.5) × $650 = $744.60

Therefore, does the proposed price appear to be fair and reasonable? Defend your determination. *Based on our Lesson 1 percentage-change lesson, this price is 14% higher than the index data indicate the price should be.*

$: $850 proposed - $744.60 estimated = $105.40 higher
%: $105.40 / $744.60 = 14.1% over the $744.60 estimate

With such variance, we should conduct additional market research or fact-finding, pursue an explanation for the variance, and then make the determination of price reasonableness. Two specific learning points: a) price indexing does not automatically enable us to determine a price to be fair and reasonable; b) the concept of "analyzing and explaining variance" is common in contract pricing, is emphasized again in the next lesson, and in even more detail in CON 270.

Lesson 3 -- ELO 2.02 Given a set of data, analyze Data Shape, Center, Spread and Trend characteristics.

The enabling learning objectives of this lesson:

- Given a set of data, describe the measures of centrality and dispersion
- Gain understanding of how to graph and interpret simple data sets related to contract pricing
- Gain understanding of how to formulate decisions based on fundamental conclusions drawn from graphing and pictorial data analysis

Introduction

This lesson presents fundamental concepts of how to graphically display and analyze data. In addition, the lesson presents how to recognize data patterns for predictive purposes.

Fundamental Measures of Centrality. One characteristic of a set of numbers is the middle point of the data. This can be calculated in a couple of different ways:

The **Mean** is calculated by adding all the numbers and dividing by the number of values.

The **Median** is the middle value of the numbers listed in rank order. If there is an even number of values, the median is the mean of the middle two numbers.

The **Mode** is the most frequent value in a set of data.

Shape of the Data Set. Plotting of data may give insight into a characteristic of the data known as shape.

Data with a **Normal** shape will have the same mean and median. In the normal case, data will be balanced around the middle.

Data might be **Skewed** either to the right (positive, with the low "tail" pointing to the right) or left (negative, with the low "tail" pointing to the left). In skewed data, a small number of extremely positive or negative values caused the mean to be either higher or lower than the median. When graphed, this produces a tail of data to the right or left.

Data distribution may be **Bi-Modal or Multi-Modal**. These means that there is more than one cluster of occurrences of data values along the continuum that when graphed appears as multiple peaks of data.

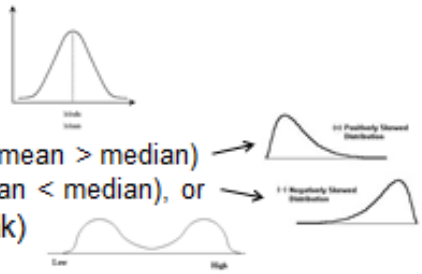
Spread of the data is generally concerned with the range from low to high value and can give indication to the expected extent of the values possible.

Data Trends can also be determined. Trends result when a separate influence or variable is determined to influence data. Once the relationship between this separate variable and the data set of interest is established, knowing the value of the separate variable will help determine characteristics of the data set.

As a contracting professional, you should become familiar with the types of scenarios that will be worked in class, as each one describes a unique aspect of data analysis in terms of Center, Shape, Spread, and Trend.

Center, Shape, Spread and Trend

- Center
  - Mean (arithmetic average), and
  - Median (middle value)
- Shape
  - Normal (mean = median), or
  - Skew (mean ≠ median),
    - Right Skew (tail to the right, mean > median)
    - Left Skew (tail to the left, mean < median), or
  - Bi-modal (more than one peak)
- Spread
  - Minimum (Min); smallest value in data set
  - Maximum (Max); largest value in the data set
  - Range; Max minus (-) Min
- Trend
  - Will an independent variable help explain variation (spread)
  - Size, Quality, Performance



Center, Shape, Spread and Trend

- Average:
  - a single value (as a mean, mode, **or** median) that summarizes or represents the general significance of a set of unequal values (Websters On-line)
  - A number that typifies a set of numbers (The Free Dictionary)
- Arithmetic Mean:
  - Sum of the values in a set of numbers divided by the number of values (observations) in that set
- Median:
  - The middle value in a set of numbers
    - For an even number of observations – the sum of the middle two values divided by two (2)
  - 50% of the observations are above the median and 50% are below the median
  - When equal to the arithmetic mean, the distribution is often called "normal" (aka – the bell shaped curve)

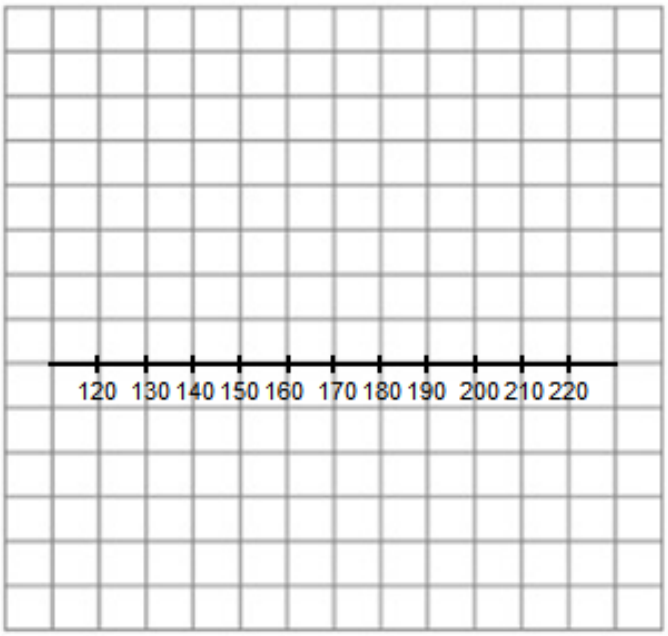
Center, Shape, Spread and Trend

- **Scenario #1**: Historical prices paid for radios has been collected from market research and properly normalized (i.e. adjusted for content, economics, quantity, etc.) as follows: $166, $154, $176, $164, $155, $162, and $171.
  - Calculate the mean by totaling the values and dividing by the number of prices.
  - Rank order the data, highest to lowest, and:
    - Find the maximum value.
    - Find the minimum value.
    - Find the median (middle) value.
    - Calculate the range (subtract minimum from maximum).
  - Develop a histogram by shading in a square for each observed price you find in the bin annotated on the graph paper.
  - Draw a vertical line through the bin that would contain the mean and the median and label it appropriately.

Scenario #1

Rank Ordered

166
154
176
164
155
162
171



Mean:
Median:
Minimum:
Maximum:
Range:

Center, Shape, Spread and Trend

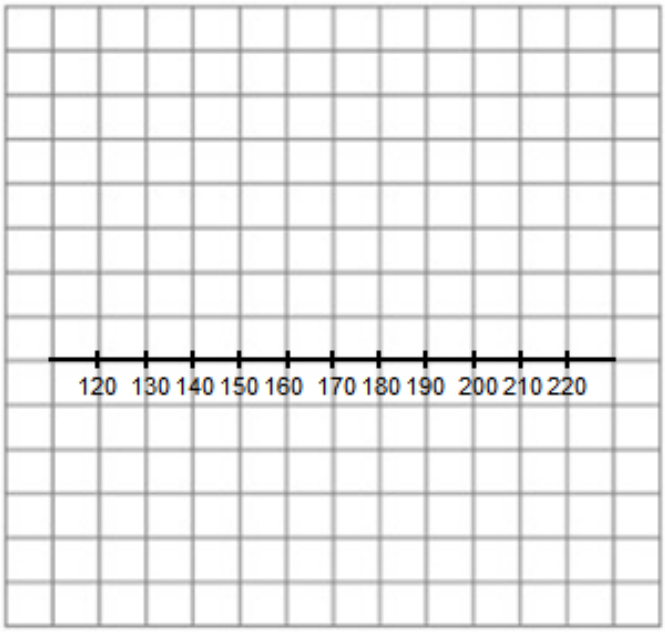
- **Scenario #2**: Historical prices paid for radios has been collected from market research and properly normalized (i.e. adjusted for content, economics, quantity, etc.) as follows: $163, $171, $148, $158, $165, $131, $182, and $194.
  - Calculate the mean, median, and range while also identifying the minimum and maximum values.
  - Develop a histogram by shading in a square for each observed price.
  - Draw a vertical line through the bin that would contain the mean and the median and label it appropriately.

Answer the following for each:	Scenario #1	Scenario #2
Typical Price?		
Used mean or median?		
In which (#1 or #2) are you more confident?		
Range?		

Rank Ordered

163
171
148
158
165
131
182
194

Scenario #2



Mean:
Median:
Minimum:
Maximum:
Range:

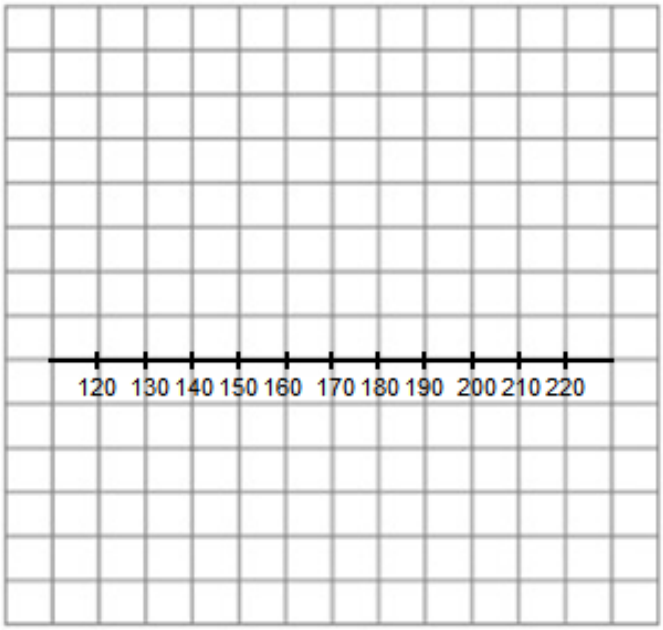
Center, Shape, Spread and Trend

- **Scenario #3**: Historical prices paid for radios has been collected from market research and properly normalized (i.e. adjusted for content, economics, quantity, etc.) as follows: $132, $149, $209, $133, $141, $132, $219, $131, $132, $199, $189, $131, and $131 .
  - Calculate the mean, median, and range while also identifying the minimum and maximum values.
  - Develop a histogram by shading in a square for each observed price.
  - Draw a vertical line through the bin that would contain the mean and the median and label it appropriately.
- Based on the appearance of your histogram, would you say that your distribution is:
  1. Left skewed (tail to the left), or
  2. Normal, or
  3. Right skewed (tail to the right), or
  4. Bimodal?
- Is your mean equal to, greater than, or less than your median?
- Which "average" value, mean or median, best typifies your sample?

Scenario #3

Rank Ordered

132
149
209
133
141
132
219
131
132
199
189
131
131



Mean:
Median:
Minimum:
Maximum:
Range:

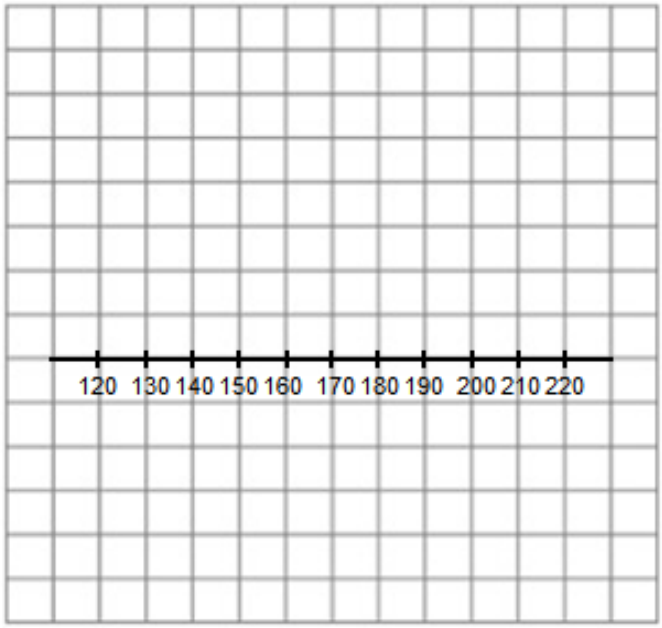
Center, Shape, Spread and Trend

- **Scenario #4**: Historical prices paid for radios has been collected from market research and properly normalized (i.e. adjusted for content, economics, quantity, etc.) as follows: $203, $122, $202, $192, $201, $131, $202, $201, $191, $112, $141, $201, and $202.
  - Calculate the mean, median, and range while also identifying the minimum and maximum values.
  - Develop a histogram by shading in a square for each observed price.
  - Draw a vertical line through the bin that would contain the mean and the median and label it appropriately.
- Based on the appearance of your histogram, would you say that your distribution is:
  1. Left skewed (tail to the left), or
  2. Normal, or
  3. Right skewed (tail to the right), or
  4. Bimodal?
- Is your mean equal to, greater than, or less than your median?
- Which "average" value, mean or median, best typifies your sample?

Rank Ordered

203
122
202
192
201
131
202
201
191
112
141
201
202

Scenario #4



Mean:
Median:
Minimum:
Maximum:
Range:

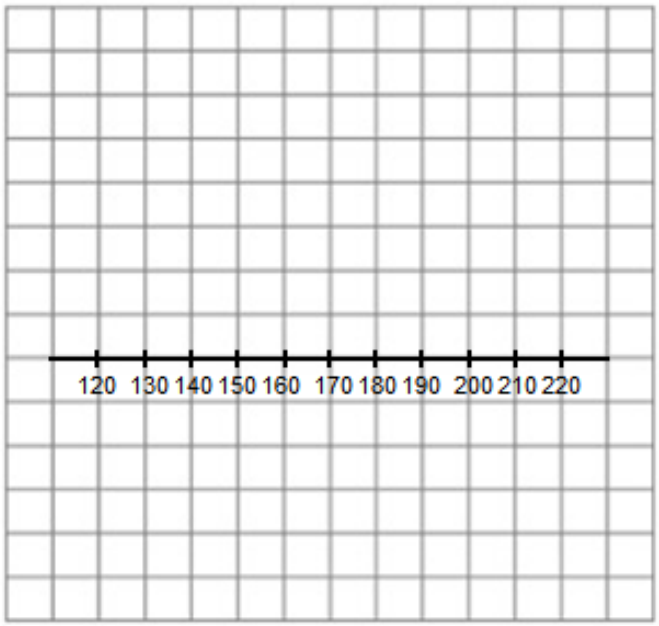
Center, Shape, Spread and Trend

- **Scenario #5**: Historical prices paid for radios has been collected from market research and properly normalized (i.e. adjusted for content, economics, quantity, etc.) as follows: $149, $188, $154, $187, $144, $196, $143, $179, $182, $131, and $162.
  - Calculate the mean, median, and range while also identifying the minimum and maximum values.
  - Develop a histogram by shading in a square for each observed price.
  - Draw a vertical line through the bin that would contain the mean and the median and label it appropriately.
- Based on the appearance of your histogram, would you say that your distribution is:
  1. Left skewed (tail to the left), or
  2. Normal, or
  3. Right skewed (tail to the right), or
  4. Bimodal?
- Which "average" value, mean or median, best typifies your sample?

Scenario #5

Rank Ordered

149
188
154
187
144
196
143
179
182
131
162



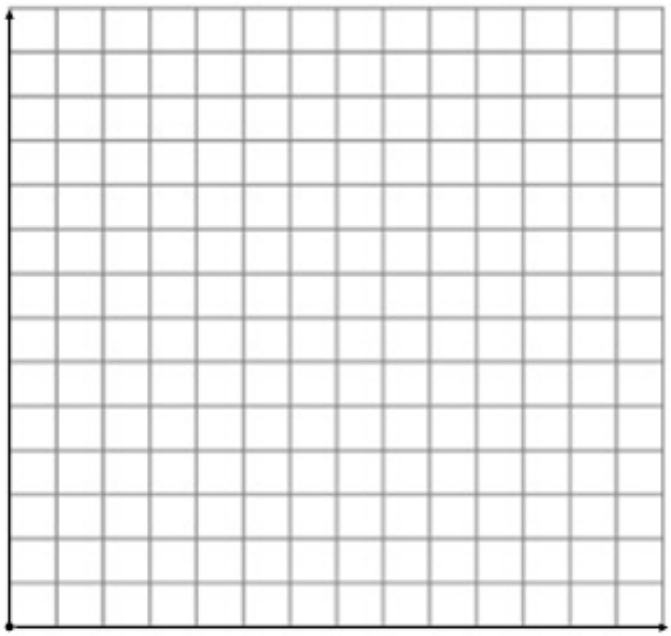
Mean:
Median:
Minimum:
Maximum:
Range:

Is there a Trend?

- **Scenario #5 Revisited**:
- Recall what we said about a trend
  - Will an independent variable help explain variation (spread)
  - Size, Performance
- Would considering whether a trend is at play perhaps help with Scenario #5?
- Can you think of some size, quantity, or performance attributes of radios that might help explain variations in price?
- But, weren't those considerations already incorporated when we collected and normalized the data?
  - What was the data normalized (adjusted) for anyway?
- Using whatever you came up with as a possible size, quantity, or performance attribute you came up with - what relationship do you think it has to price?
  - As your attribute increases, would you expect the price to increase or decrease?
- Using the graph below where Price has already been labeled as your vertical axis, label the horizontal axis with the size, quantity, or performance attribute you selected and draw the relationship freehand on the graph.

Scenario #5 revisited

Price $



Explanatory attribute: _____

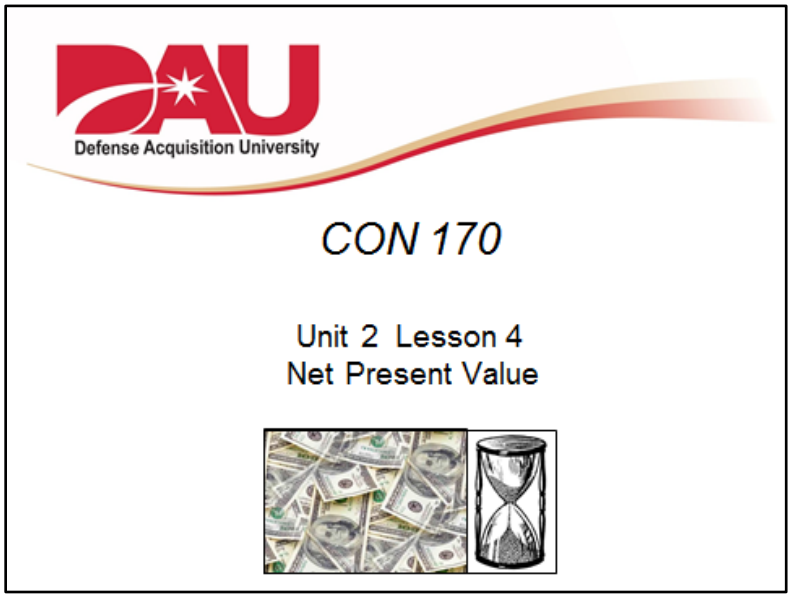
Analysis Concepts Learned

- When the mean and median are close together, this could be a sign the distribution is normal – unless it is a multi-modal distribution.
- With small data sets there may or may not be an official "mode" thus graphical analysis is helpful in identifying distribution types.
- Even with equal means and medians under normal distributions, wider distributions (larger ranges) will leave you less confident (Scenario # 1 vs. #2).
- Means and medians may be the same or very close under both normal and multi-modal distributions, but when they are different the distribution could be skewed one way or the other (eg. Scenarios #3 and #4).
- With distributions skewed right, the mean is more affected by the extreme values in the tail to the right and will be larger than the median (e.g. Scenario #3).
- With distributions skewed left, the mean is more affected by the extreme values in the tail to the left and will be less than the median.
- The mean often used as the best "average" to typify the data, but this is true when the data are normally distributed (in which case it is close to the median anyway).
- When dealing with skewed data, the median may be the best "average" to use to typify the data.
- When finding skewed or multi-modal data, or data with large spread (high ranges, variability) an investigation into the data to find if it has indeed been properly normalized (adjusted) for content, quantity, and economics is called for.
- After skewed data, multi-modal data, or data with greater variability, are investigated and corrected to be properly normalized for content, quantity, and economics; additional size and/or performance may be hypothesized to logically affect price to consider whether a trend might be considered.

The purpose of this lesson has been to highlight some basic ways of analysis simple sets of data. To recap, key values are easily calculated:

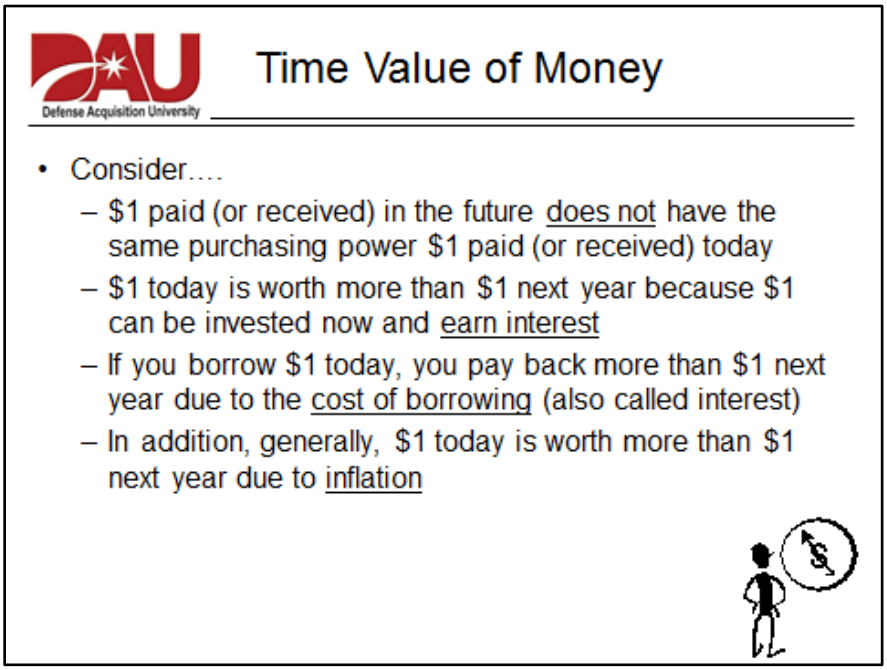
- **Mean** = Sum of the observations divided by the number of observations.
- **Median** = After rank ordering the observations - the middle value or the average of the middle two if there is an even number of observations
- **Range** = Subtraction of the Minimum from the Maximum
- **Distribution** of a set of numbers can be described in several different ways:
  - The Center can be described by either mean, median, and mode
  - Shape is commonly categorized into four common distributions: normal, right skew (tail to the right), left skew (tail to the left), or multi-modal
  - Spread can be basically described by the range calculating the difference between the maximum and minimum
  - Trend: Even with data properly normalized for content, quantity, and economics; certain size and/or performance attributes may also help explain prices when faced with wider spreads (high range) and/or multi-modal

Lesson 4 -- ELO 2.03 Given a set of data calculate the Net Present Value of the given data.

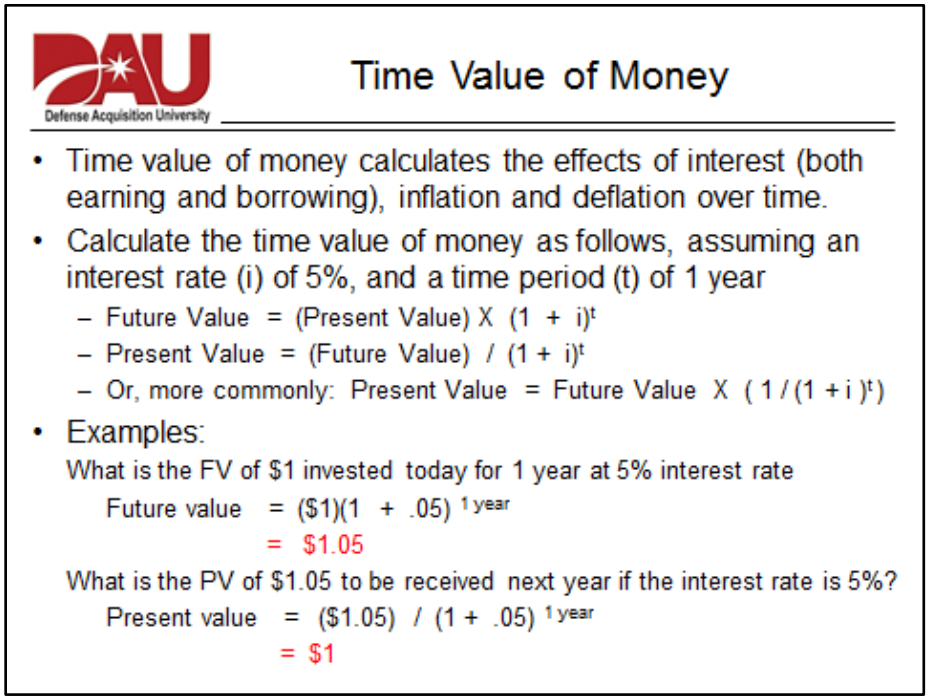


Introduction

This objective of this lesson is to analyze a set of cash flow data, and calculate the Net Present Value. First, the lesson explains the foundational concept of the time value of money. Then, the lesson presents the process for calculating present value, and concludes with calculating Net Present Value.



Time Value of Money. To introduce the Time Value of Money concept, imagine putting $1 in a savings account for one year, at an interest rate of 5%. How would we calculate the future value of the $1 at the end of the year?



Example 1. With the same process, calculate the future value of $150 after 3 years at 5% interest. (Hint: the FV is $150, i is .05, and t is 3).

$$FV = PV \times (1 + i)^t$$

From Time Value of Money to Present Value Analysis. Next, based on the time value of money concept, we can compare the present value of future cash flows. This is accomplished through present value analysis.

An example of present value analysis: A car dealer offers to sell you a car for $21,000 cash, or offers "free financing" for $21,000 due in 1 year. Which is a more attractive offer? Certainly, you'd prefer to pay the $21,000 later, so you could invest the money for 1 year and earn interest.

Then, to persuade you to "drive a car home today," he offers to sell you the car for $20,000 cash on delivery (now), or sell it for $21,000 if you pay the entire amount 1 year from now, with his special 5% financing. Which deal is best? To find out, we must calculate and compare the present value of each alternative.

- The offer to pay $20,000 now has a present value (PV) of $20,000, for it will be paid "now."

- The other offer has a future value of $21,000. What is its PV?
  To calculate the PV, use the same formula as we used in the earlier examples:

  $$\mathbf{FV = PV \times (1 + i)^{t}}$$

  In this example, we have the future value of $21,000, and we are trying to calculate the present value (PV). Therefore, we manipulate the equation as follows:

  $$\mathbf{PV = FV \ / \ (1 + i)^{t}}$$

  which is the same as:

  $$\mathbf{PV = FV \times [\,1 \ / \ (1 + i)^{t}\,]}$$

- To calculate the present value, insert the values as follows
  - The "future value" of the payment, FV, is $21,000
  - The "interest rate" at which we would borrow money, i, is 5%
  - The timeframe we would borrow the money, t, is 1 year.

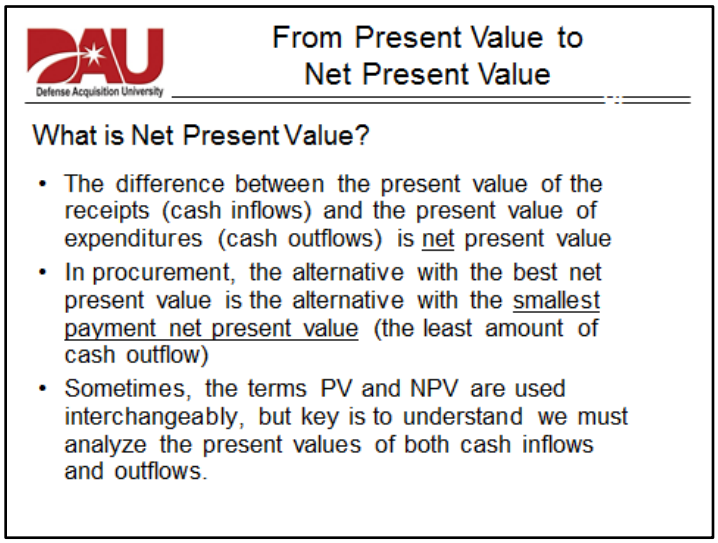
- Finally, insert the values in to the PV equation, and solve for PV:

  $$PV = \$21{,}000 \times [\,1 \ / \ (1 + .05)^{1yr}\,]$$
  $$PV = \$21{,}000 \times [\,1 \ / \ 1.05\,]$$
  $$PV = \$20{,}000$$

By comparing the present values of each offer, we see these two pricing arrangements are essentially identical, for they both have the same "present value" of $20,000.

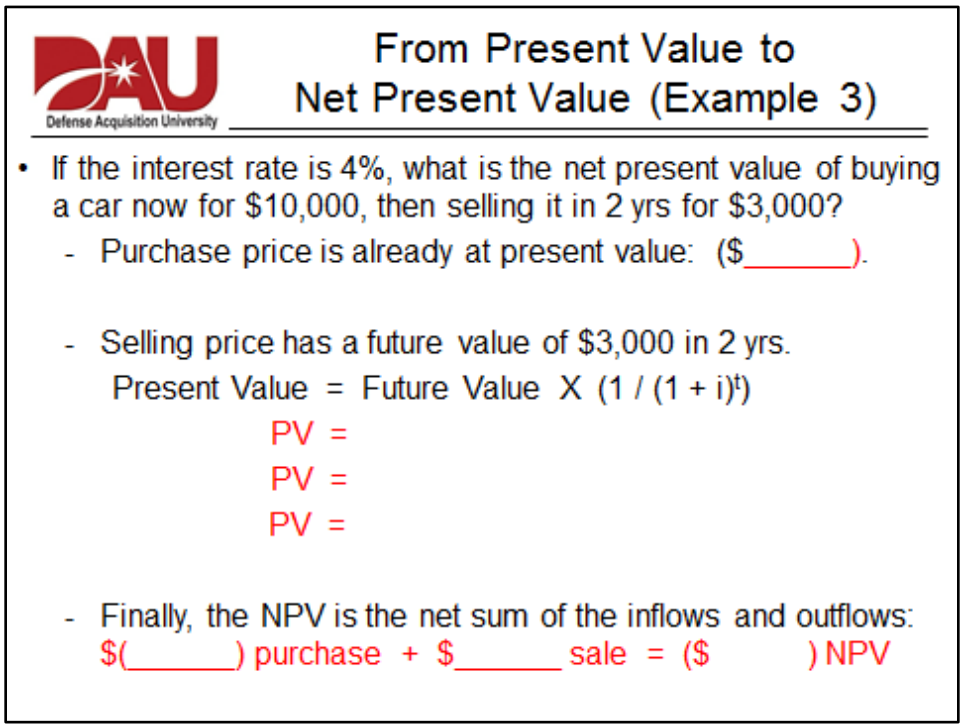
Example 2. In the space below, calculate the present value of a future cash outflow. If the interest rate is 4%, what is the present value of $300 to be paid 3 years from today? (Round to the nearest dollar). ***Hint: calculate the FV with a formula we used before.***

Net Present Value Analysis. With an understanding of present value analysis, it is time to learn Net Present Value analysis, or NPV.

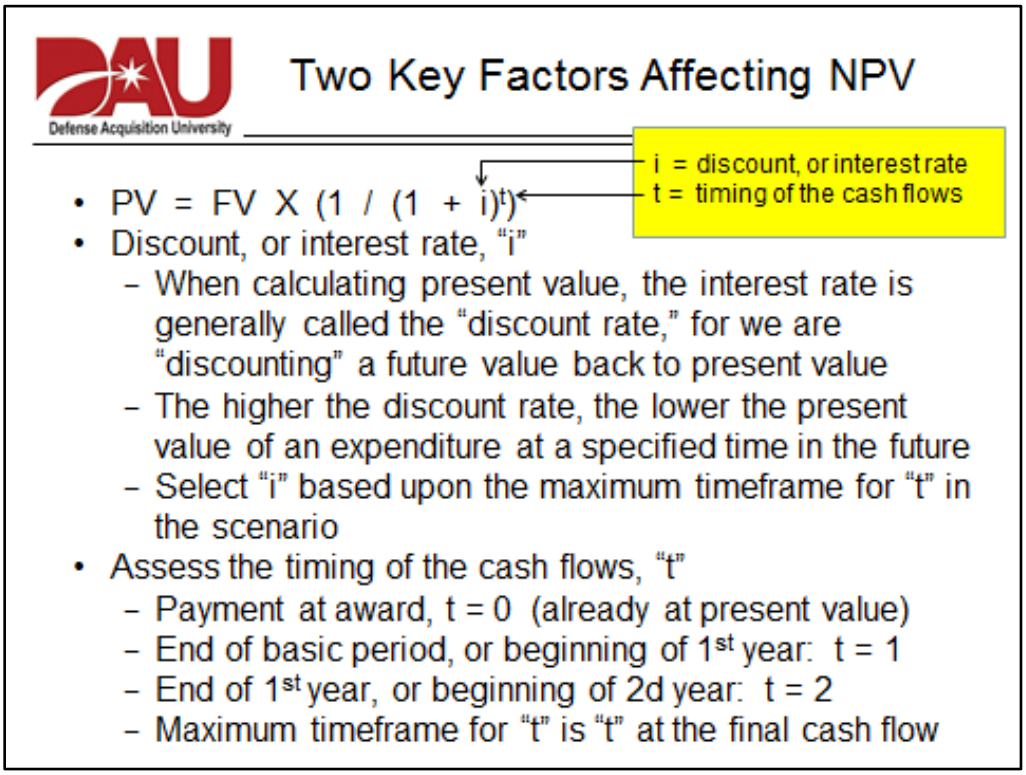


As we learn Net Present Value (NPV), recognize the terms PV and NPV are sometimes used synonymously; however, it is important to recognize NPV analysis includes calculating the PV of both cash inflows and cash outflows—thus, the term "net" present value. To calculate NPV, figure out the present value of a cash outflow, then add the present value of the cash inflow.

Example 3. Calculate the NPV of the following scenario: you pay $10,000 to purchase (cash outflow) a car now, and with a plan to sell the car (cash inflow) in after 2 years for $3,000. The interest rate is 4%.



In conducting NPV calculations for Government acquisitions, there are two key factors we must consider: the interest rate (generally called a "discount rate" for NPV) and the timeframe of the scenario. As we have already seen, these variables are represented by "i" and "t".



How to Select the "i" and "t" values for NPV in DoD Acquisitions

Per FAR 32.205(c), select from rates provided by the Office of Management and Budget: OMB Circular A-94 Appendix C. OMB rates reflect standard assumptions regarding inflation and cost of borrowing money, and provide consistency in analysis throughout Federal Govt. Discount rates are posted on the the OMB website: http://www.whitehouse.gov/omb/circulars_a094/a94_appx-c

Per FAR 32.205(c)(4), use the nominal discount rate. The nominal interest rate reflects the real inflation rate plus a risk factor associated with normal lending. We use the nominal rate because it more closely reflects the business environment where contractors borrow and repay money in the course of doing business. Contractors commonly refer to this as their "weighted average cost of capital."

The "real" interest rate accounts for the effect of inflation on an investment, and therefore indicates the growth rate of purchasing power earned from an investment. For example, if you earn 5% in one year, but the inflation rate is 3.5%, the "real" interest rate earned is 1.5%.

The following slide presents a comparison of the Real Discount Rate and the Nominal Discount Rate. Notice, the nominal discount rates are positive for all years. However, in 2014, the three

shortest- term real discount rates are actually *negative*! As described in the slide above, a negative real discount rate indicates the expected inflation rate will be higher than the nominal interest rate.

Per FAR 32.205(c)(4), we must use the nominal discount rate in our NPV assessments.

Discount Rates per OMB A-94 Appendix C
(Latest rates posted as of Jan 2013)

Maturity in Years	Real Discount Rate	Nominal Discount Rate
< 3-years	-1.4%	0.5%
5-years	-0.8%	1.1%
7-years	-0.4%	1.5%
10-years	0.1%	2.0%
20-years	0.8%	2.7%
30-years	1.1%	3.8 %
(Scenario Timeframe)	(Cost of Borrowing)	(Cost of Borrowing & Inflation) Per FAR 32.205(c)(4): use the nominal discount rate

Remember: in conducting NPV analysis, select the discount rate based on maximum timeframe for "t" in scenario. If the maximum timeframe is less than or equal to 3 years, use the 3 year nominal rate. If the maximum t-value is beyond 3 years, FAR 32.205(c)(4) gives guidance: "Where the period of proposed financing does not match the periods in the OMB Circular, the interest rate for the period closest to the finance period shall be used."

Simplifying the NPV calculation.

With an understanding of each element in the NPV equation, we can simplify our calculations as shown in the 2 following slides. The first slide explains how to simplify the PV equation by substituting the expression "$[1 / (1 + i)^t]$" with an element called the "Discount Factor," or DF. Thus,

$$\mathbf{PV = Future\ Cash\ Flow \times DF.}$$

Or,

$$\mathbf{PV = CF \times DF}$$

Simplifying the Net Present Value Calculation

- The equation for calculating Net Present Value is based upon:

  Present Value = Cash Flow X $(1 / (1 + i)^t)$

- To simplify the calculation of $(1 / (1 + i)^t)$, we can select the appropriate "Discount Factor (DF)" from the "Discount Factor Table" in the next slide, then substitute the DF in place of $(1 / (1 + i)^t)$

- To select the appropriate DF from the Discount Factor Table, we must determine the appropriate "t" and "i" values.

- Finally, the Net Present Value calculation is based upon:

  Present Value = Cash Flow X Discount Factor

  or

  PV = CF X DF

Then, we can simplify our arithmetic by plugging in a value for DF from the next slide, "The Discount Factor Table." For example, if our NPV scenario included a maximum timeframe of 3 years, and the current discount rate is .5%, we can simply select the appropriate Discount Factor from the table below: .9851. This is much simpler than calculating "$[1 / (1 + i)^t]$"!

Discount Factor Table

Per OMB Circular A-94's 3 and 5-year Discount Rates
(assume Nominal Rate, current as of 20 Jan 2013)

	Scenario Length			
	3 years		5 years	
Year, "t"	Disc Rate	Disc Factor	Disc Rate	Disc Factor
0	0.5%	1	1.1%	1
0.5	0.5%	0.9975	1.1%	0.9945
1	0.5%	0.9950	1.1%	0.9891
1.5	0.5%	0.9925	1.1%	0.9837
2	0.5%	0.9901	1.1%	0.9784
2.5	0.5%	0.9876	1.1%	0.9730
3	0.5%	0.9851	1.1%	0.9677
3.5			1.1%	0.9624
4			1.1%	0.9572
4.5			1.1%	0.9520
5			1.1%	0.9468

We'll use this table for our examples!

At this point, we have learned all the elements for calculating an NPV problem as DoD buyers. Because NPV analysis generally requires calculations of cash inflows and outflows over several time periods, CON 170 NPV problems will be presented using tables like the one below.

Exercise 1. Using the table below, solve the following NPV problem. You are considering the purchase of a vehicle now for $10,000. You will sell it in 2 years for $3,000. Assume you will use the current discount rate as published by OMB Circular A-94. What is the NPV of this arrangement?

Calculating Net Present Value Exercise 1

- To simplify the NPV calculations, use a table as follows:

Timeframe t		Cash Flow CF	Discount Rate i	Discount Factor DF (per t and i)	PV CF X DF
TOTAL NPV					

Using the table above, solve the NPV problem one step at a time, and fill in the blanks below:

- Step 1: Identify the timeframes and amounts of the cash flows:
  - First, the $10,000 payment, with t = 0 since it is at the present time.
  - Next, the sale of the vehicle means a $3,000 cash inflow 2 years after the purchase; therefore, t = _____

- Step 2: Identify the correct discount rates:
  - Determine the maximum timeframe for this scenario--2 years. Therefore, we will use the information under the "3-year Scenario" columns in the <u>Discount Factor Table</u>.
  - Therefore, our discount rate will be _____ for all cash flows in this scenario.

- Step 3: Identify the correct Discount Factor, per the Discount Factor Table
  - For the $10,000 payment: under the "3 year Scenario Length" data, at t = __ the discount rate is _____. Per the Discount Factor Table, the DF is 1.0000

- For the $3,000 sale: under the "3 year Scenario Length" data, at t = ___, the discount rate is _____. Per the Discount Factor Table, the DF is _____

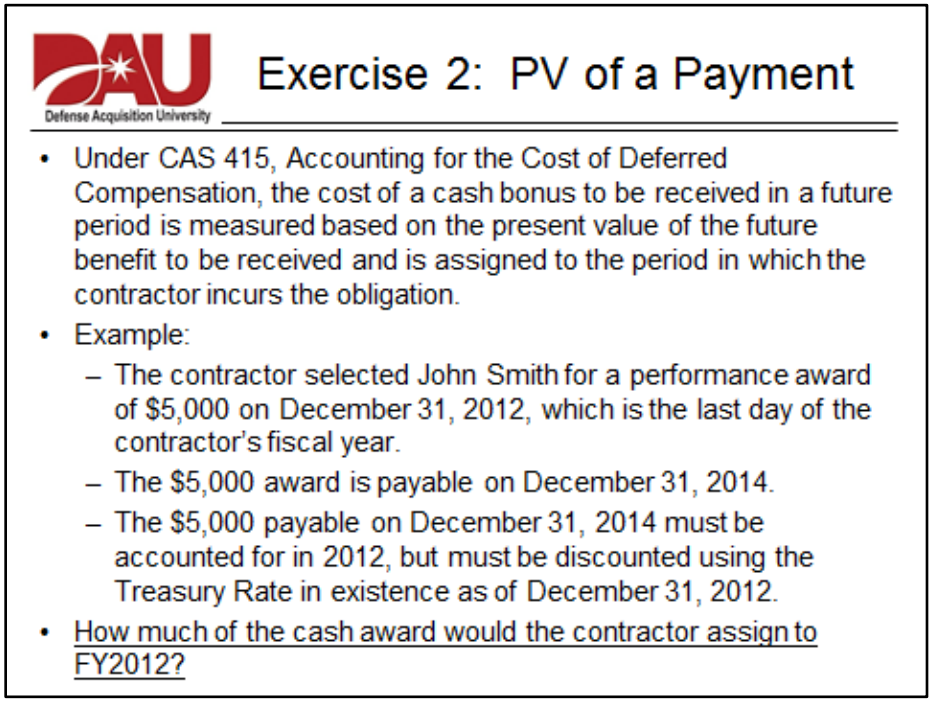
  Student Note: for a scenario over 3 years, but less than or equal to 5 years, use the information under the "5 year Scenario Length" heading in the Discount Factor Table. The OMB website includes scenarios even beyond 5 years, but we will not use them in this class.

- Step 4: calculate the PVs of each cash flow (CF × DF), and sum them to find the NPV.
  - For the $10,000 payment: ($10,000) × _____ = _____
  - For the $3,000 sale: $3,000 × _____ = _____
  - Take special care to use negative numbers for cash outflows, such as payments (placed in parentheses), and positive numbers for cash inflows, such as sales income (not placed in parentheses).
  - The sum of the PVs is _____, which is the NPV.

- Step 5: When evaluating two or more pricing arrangements, compare each, and determine which NPV offers the best value.

Now that we are familiar with fundamental concepts of net present value, we will complete several real-world problems using net present value analysis, using the 5-step process stated above.

Exercise 2. Our first example is based upon a scenario where DCAA, DCMA, and a PCO are analyzing the area of deferred compensation.



Exercise 2: PV of a Payment

Timeframe t		Cash Flow CF	Discount Rate i	Discount Factor DF (per t and i)	PV CF X DF
TOTAL NPV					

Using the table provided, solve the NPV problem one step at a time, and fill in blanks below:

- Step 1: Identify the timeframes and amounts of the cash flows:
  - The $5,000 payment, will not be made for 2 years. Therefore, t = _____
- Step 2: Identify the correct discount rates:
  - The maximum timeframe for this scenario--2 years. Therefore, we will use the information under the "____-year Scenario" columns in the Discount Factor Table.
  - Therefore, our discount rate will be ____% for all cash flows in this scenario.
- Step 3: Identify the correct Discount Factor, per the Discount Factor Table
  - For the $5,000 payment: under the "3 year Scenario Length" data, at t = ___ the discount rate is _____%. Per the Discount Factor Table, the DF is _____
- Step 4: calculate the PVs of each cash flow (CF × DF), and sum them to find the NPV.
  - For the $5,000 payment: ($5,000) × _____ = _____
  - In this scenario, we only have one cash flow. Therefore, the sum of all the PVs is _____ which is the NPV.
- Step 5: When evaluating two or more pricing arrangements, compare each, and determine which NPV offers the best value. (not applicable for this scenario)

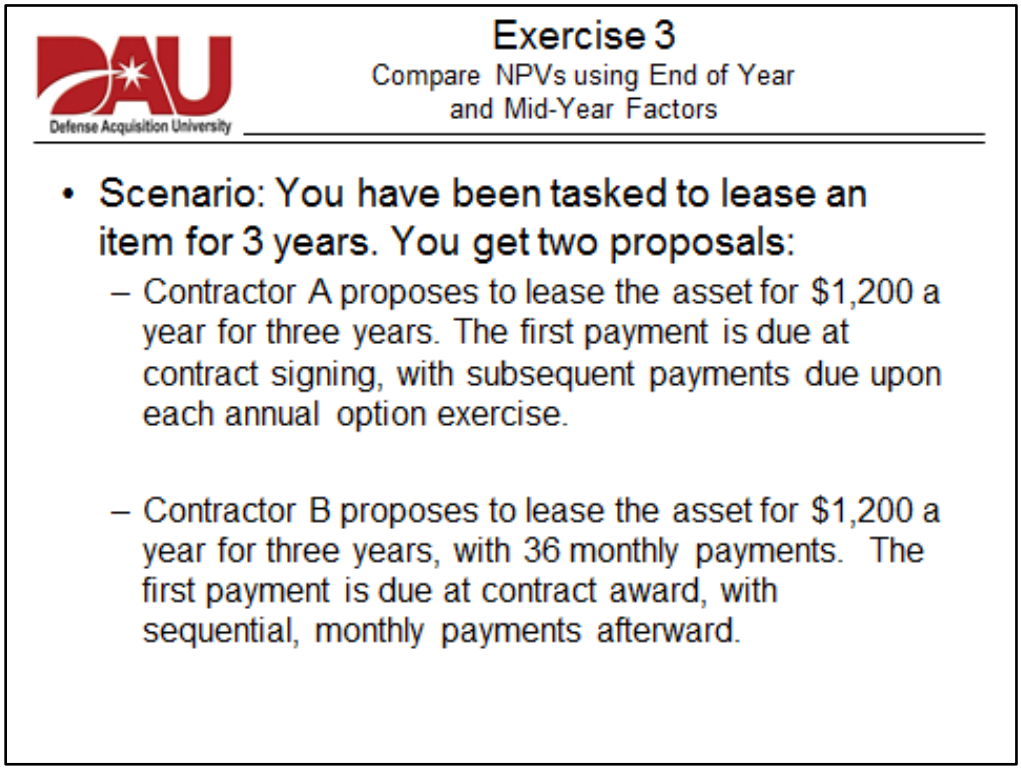
Discussion. Based on the calculations:

To comply with CAS 414, even though the award is for $5,000, the company assigns for _____ as a 2012 expense.

To pay for Mr. Smith's award, the company will invest _____ at the end of December 2012, in order to pay Mr. Smith $5,000 at the end of December in 2014.

This means, Mr. Smith would consider a payment of _____ at the end of 2012 essentially equal to a payment of $5,000 at the end of 2014.

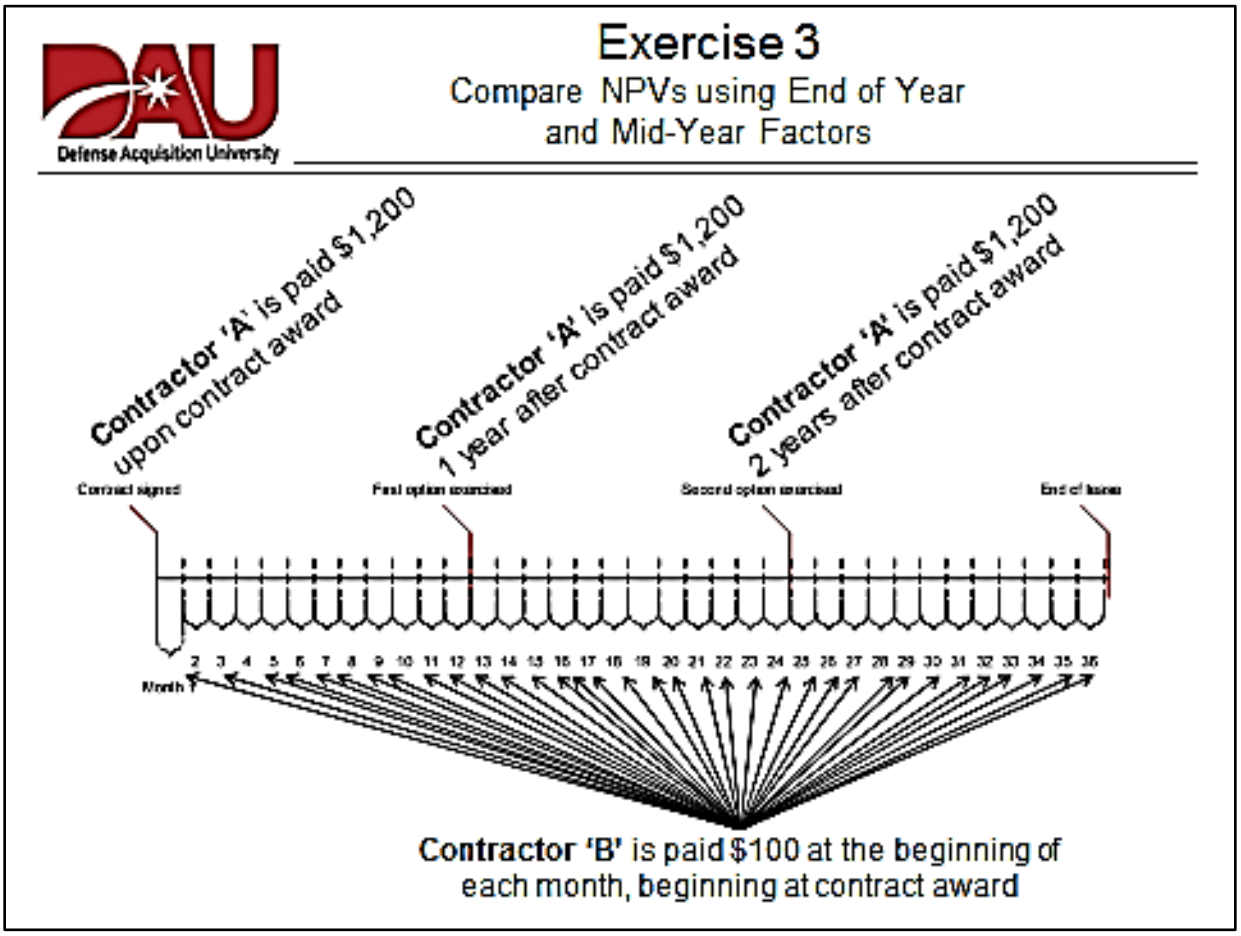
Exercise 3. In this example, you must examine 2 different proposals and compare their NPVs. This example introduces how we must calculate NPV with either "end of year" Discount Factors, or "mid-year" Discount Factors. In addition, this example illustrates how two proposals with the same face value can have different NPVs based on the timing of cashflows.



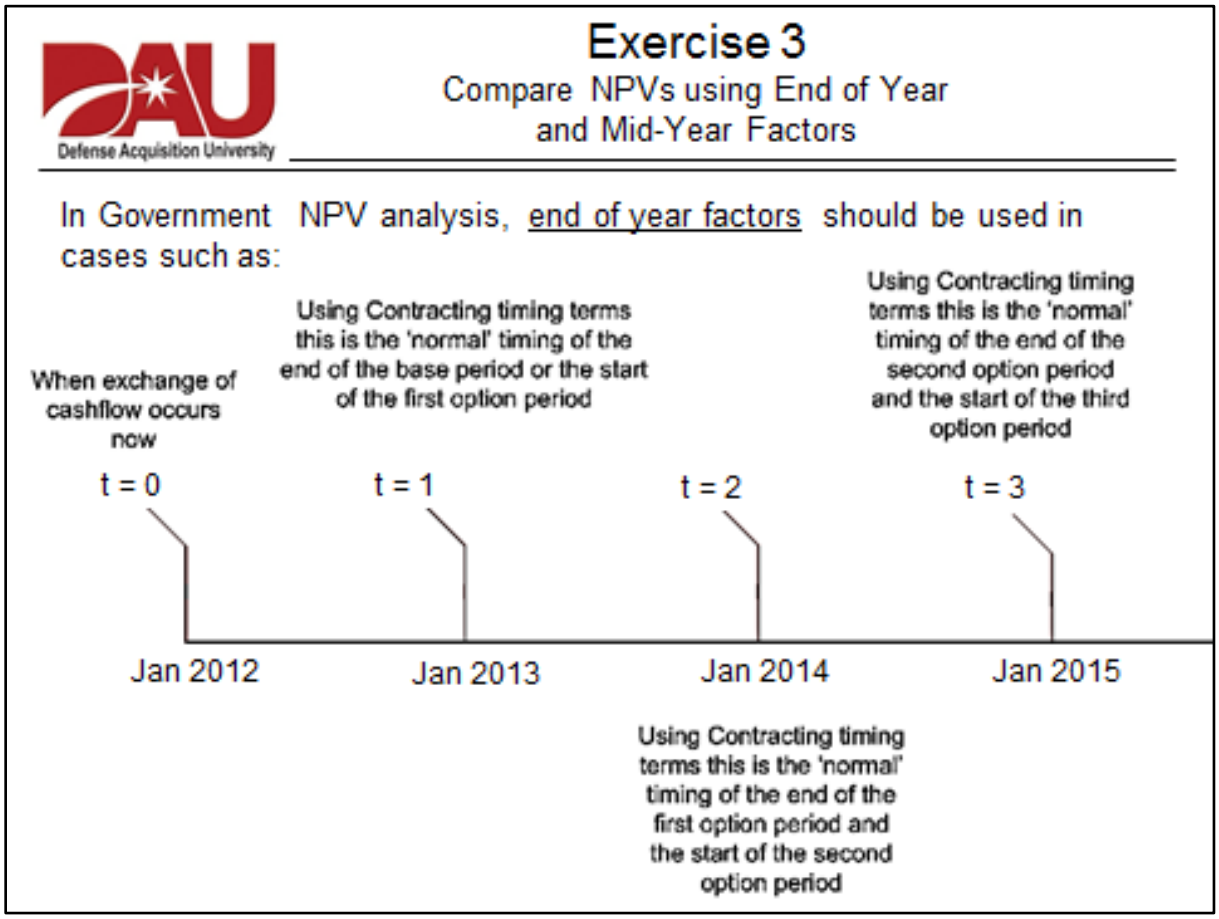
In this scenario, we calculate the NPVs of each offer, then compare them. Before beginning our 5-step process, notice the differences in the proposed cash flows. Contractor A proposes one payment each year at the beginning of each year, for a total of 3 payments. Contractor B proposes one payment each month for 3 years, for a total of 36 payments.

These different cash flow schedules proposed by Contractors A and B introduce us to "end of year" and "mid-year" Discount Factors. The slide below graphically depicts the cash flow of Contractor A

compared to Contractor B. From what we have learned so far, would you prefer a payment schedule where you received the entire amount at the beginning of the year, or would monthly payments be more advantageous?



For Contractor A, each of the cash flows (payments) is made at the beginning of each year. When a cash flow is made at the very beginning (or the very end) of a year, "t" is equal to a whole number. The Discount Factors that accompany such "t" values are known as "<u>end of year</u>" Discount Factors. The chart below is based upon the Contract Pricing Reference Guide, Vol 2, Chapter 9, and explains when to use "end of year" Discount Factors in solving NPV problems.



As the chart indicates, analysts should use "end of year" Discount Factors when cash flows occur annually, at the beginning or end of each year. In examining Contractor A's proposed cash flow of 3 annual payments, it is appropriate to use "end of year." Discount Factors.

Contractor A's proposed cash flow is 3 annual payments, at t = 0, t = 1, and t = 2. The chart below highlights the "end of year" Discount Factors in our Discount Factor Table. Notice, the "t" values which are whole numbers have "end of year" Discount Factors. Therefore, to calculate the PV of Contractor A's payments, we would use the 3 "end of year" Discount Factors highlighted in the slide below.

Discount Factor Table

Per OMB Circular A-94's 3 and 5-year Discount Rates
(assume Nominal Rate, current as of 15 Jan 2013)

For Offeror A, use "end of year" Discount Factors

	Scenario Length			
	3 years		5 years	
Year, "t"	Disc Rate	Disc Factor	Disc Rate	Disc Factor
0	0.5%	1	1.1%	1
0.5	0.5%	0.9975	1.1%	0.9945
1	0.5%	0.9950	1.1%	0.9891
1.5	0.5%	0.9925	1.1%	0.9837
2	0.5%	0.9901	1.1%	0.9784
2.5	0.5%	0.9876	1.1%	0.9730
3	0.5%	0.9851	1.1%	0.9677
3.5			1.1%	0.9624
4			1.1%	0.9572
4.5			1.1%	0.9520
5			1.1%	0.9468

We'll use this table for our examples!

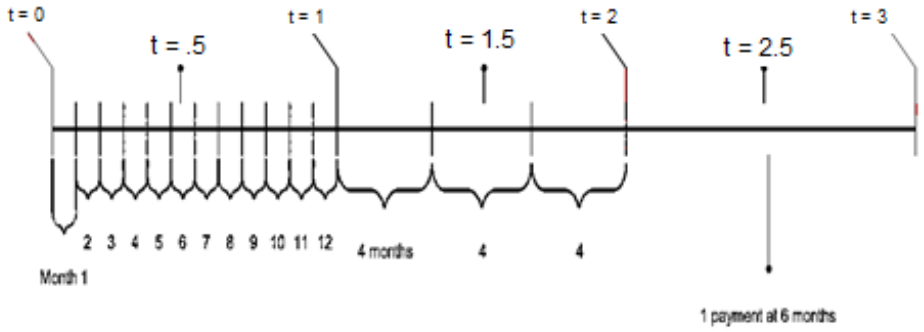
Now consider the cash flows for Contractor B's offer. With proposed cash flow of 36 monthly payments, should we execute 36 calculations to determine the PV of each payment? Though some members of the pricing career field may enjoy doing so, they have an even simpler approach. The chart below is also based upon the Contract Pricing Reference Guide, Vol 2, Chapter 9, and introduces the "mid-year" Discount Factors for solving NPV problems.

Exercise 3

Compare NPVs using End of Year and Mid-Year Factors

In Government NPV analysis, mid-year factors should be used in cases such as:

1. Consecutive monthly payments
2. Consecutive payments every 4 months
3. A single, mid-year payment



In the chart above:

The initial period (when t = 0) includes 12 monthly payments. Rather than sum the present values of each of the 12 payments, where t would equal 0, then 1/12, then 2/12, then 3/12…all the way to 11/12, we can simply calculate the present value of the entire year's cash flows using the "mid-year" t-value of .5. From the Discount Factor Table below, this would equate to a "mid-year" DF of .9975.

The next period in the chart above (when t = 1) includes 1 payment every 4 months, which equates to 3 payments in the year. Rather than sum the present values of each of the 3 payments, where t would equal 1, then 4/12, then 8/12, we can simply calculate the present value of the entire year's cash flows using the "mid-year" t-value between t = 1 and t = 2, which is t = 1.5. From the Discount Factor Table below, this would equate to a "mid-year" DF of .9925.

The final period in the chart above (when t = 2) includes 1 payment at the 6 month point. In this case, simply calculate the present value of the year's cash flow by using the t-value halfway between t = 2 and t = 3, which is t = 2.5. From the Discount Factor Table below, this equates to a "mid-year" DF of .9901.

As the chart indicates, analysts should use "mid-year" Discount Factors when an offeror proposes consecutive monthly, consecutive quarterly, or even mid-year payments. The "mid-year" Discount Factors enable us to calculate the present value of consecutive monthly payments, quarterly payments, and a single payment in the middle of a year with a single calculation, using a "t" value that ends in ".5".

In examining Contractor B's proposed cash flow of 36 consecutive monthly payments, it is appropriate to use "mid-year" Discount Factors. But do we use the "mid-year" factors and make 36 calculations? Thankfully, no. The "mid-year" Discount Factors enable us to calculate the PV of the 12 monthly payments in a year with a single calculation. For Contractor B's proposed cash flow of 36 monthly payments, we calculate the PV with 3 calculations (not 36), where t = .5, t = 1.5, and t = 2.5.

The chart below highlights the "mid-year" Discount Factors in our Discount Factor Table. Notice, the "t" values for the "mid-year" Discount Factors end in ".5". To calculate the PV of Contractor B's payments, we would use the 3 "mid-year" Discount Factors highlighted in the slide below.

Discount Factor Table

Per OMB Circular A-94's 3 and 5-year Discount Rates
(assume Nominal Rate, current thru Dec 2012)

For Offeror B, use "mid-year" discount factors

	Scenario Length			
	3 years		5 years	
Year, "t"	Disc Rate	Disc Factor	Disc Rate	Disc Factor
0	0.5%	1	1.1%	1
0.5	0.5%	0.9975	1.1%	0.9945
1	0.5%	0.9950	1.1%	0.9891
1.5	0.5%	0.9925	1.1%	0.9837
2	0.5%	0.9901	1.1%	0.9784
2.5	0.5%	0.9876	1.1%	0.9730
3	0.5%	0.9851	1.1%	0.9677
3.5			1.1%	0.9624
4			1.1%	0.9572
4.5			1.1%	0.9520
5			1.1%	0.9468

We'll use this table for our examples!

With this understanding of "end of year" and "mid-year" Discount Factors," we are ready to calculate the NPV of each offeror, and determine which is most advantageous to the Government.

Using the 5-Step process (beginning with Contractor A):

- Step 1: Contractor A Identify the timeframes and amounts of the cash flows:
  - Assess if "end of year" or "mid-year" factors are appropriate: in this case, we have payments at the beginning of each year, for 3 consecutive years.
  - _____ Discount Factors are appropriate.
  - A $1,200 payment, is made upon award. Therefore, **t = ___**
  - Another $1,200 payment is made one year later. Therefore, **t = ___**
  - A final $1,200 payment is made one year after that. Therefore, **t = ___**

Exercise 3 : Step 1
End of Year or Mid-Year Discount Factors?

- What is the timing of the 3 payments for Contractor 'A'?
  - Payment required upon contract award
  - Next payment upon end of base period, beginning of 1st option
  - Final payment upon end of 1st option, beginning of 2nd option
- Therefore, we must use (circle one)
  - End of Year or Mid-year factors?
- Cash flows occur: (insert a value for "t")
  - Contract award, t = ___
  - End of base period, beginning of 1st option, t = ___
  - End of 1st option, beginning of 2nd option, t = ___

- Step 2: Identify the correct discount rates:
  - The maximum timeframe for this scenario--3 years. Therefore, we will use the information under the "**___-year Scenario**" columns in the Discount Factor Table.
  - Therefore, our discount rate will be **___%** for all cash flows in this scenario.

- Step 3: Identify the correct Discount Factor, per the Discount Factor Table
  - First payment of $1,200, at the t and discount rate above, DF is _____
  - Next payment of $1,200, at the t and discount rate above, DF is _____
  - Final payment of $1,200, at the t and discount rate above, DF is _____

- Step 4: calculate the PVs of each cash flow (CF × DF), and sum them to find the NPV.
  - Calculate the PV of each of the three $1,200 payments in the table)
  - The sum of all three PVs is _____, which is the NPV.

Exercise 3 -- Offeror A

Compare NPVs using End of Year and Mid-Year Factors

Offeror A

Timeframe t		Cash Flow CF	Discount Rate i	Discount Factor DF (per t and i)	PV CF X DF
TOTAL NPV					

- Step 5: When evaluating two or more pricing arrangements, compare each, and determine which NPV offers the best value. *Before doing so, we must calculate the NPV of Contractor B's offer.*

Now, execute the 5-step process for Offeror B.

- Step 1: Contractor B Identify the timeframes and amounts of the cash flows:
  - Assess if "end of month" or "mid-year" factors are appropriate: in this case, we have payments at the beginning of each year, for 3 consecutive years.
  - _____ Discount Factors are appropriate.
  - Thus, the $100 monthly payments in the base year will be calculated as a single cash flow at **t = ____**
  - The $100 monthly payments in the next year will be calculated as a single cash flow using **t = ____**
  - The $100 monthly payments in the final year will be calculated as a single, $1,200 cash outflow using **t = ____**

Exercise 3 : Step 1
Identify Cash Flow Timing, Calculate the Discount Factor

- What is the timing of the 36 payments for Contractor 'B'?
  - First payment is due at contract award
  - Next 35 payments are made monthly
  - 36th (final) payment due upon end of 2nd year
- Therefore, we must use
  - End of Year or Mid-year factors?
  - Cash flows occur when t equals ___, ___, and ___

- Step 2: Identify the correct discount rates:
  - The maximum timeframe for this scenario--3 years. Therefore, we will use the information under the "**_____-year Scenario**" columns in the Discount Factor Table.
  - Therefore, our discount rate will be **____%** for all cash flows in this scenario.

- Step 3: Identify the correct Discount Factor, per the Discount Factor Table
  - First payment of $1,200, at the t and discount rate above, DF is _____
  - Next payment of $1,200, at the t and discount rate above, DF is _____
  - Final payment of $1,200, at the t and discount rate above, DF is _____

- Step 4: calculate the PVs of each cash flow (CF × DF), and sum them to find the NPV.
  - Calculate the PV of each of the three $1,200 payments in the table)
  - The sum of all three PVs is _____, which is the NPV.

Exercise 3 -- Offeror B

Compare NPVs using End of Year and Mid-Year Factors

Offeror B

Timeframe t		Cash Flow CF	Discount Rate i	Discount Factor DF (per t and i)	PV CF X DF
TOTAL NPV					

- **Step 5:** When evaluating two or more pricing arrangements, compare each, and determine which NPV offers the best value. With NPVs for both offerors, compare and select the best value. With all other terms being equal, the *lowest* NPV, which is the smallest cash outflow, is the best value.

Class Exercise

What's the NPV of each offer?

Offeror A's Net Present Value: ($_____)
Offeror B's Net Present Value: ($_____)

- Offeror ___ requires more cash outflow than Offeror ___
- Therefore, if all other terms are essentially equal, Offeror ___ is the best offer!

Discussion

Class Exercise

Conduct an NPV analysis to Determine the Best Value Between two Offerors.

Class Exercise
NPV to Determine Best Value

- Two offerors have submitted proposals for the same requirement.
  - Offeror A proposes a lease, with payments of $9,000 at the beginning of each year for 3 years, beginning at contract award, for a total of $27,000.
  - Offeror B proposes a vehicle purchase for $27,000 due at contract award, with a re-sale of $2,500 at the end of the third year.
- Determine which is the best offer for the Government based on Net Present Value Analysis.
- *(Remember the 5-step process, also considering "end of year" vs. "mid-year" Discount Factors)*

Class Exercise
NPV of Offeror A

Timeframe t		Cash Flow CF	Discount Rate i	Discount Factor DF (per t and i)	PV CF X DF
TOTAL NPV					

Class Exercise
NPV of Offeror B

Timeframe t		Cash Flow CF	Discount Rate i	Discount Factor DF (per t and i)	PV CF X DF
TOTAL NPV					

Class Exercise
What's the NPV of each offer?

Offeror A's Net Present Value: ($ _____)
Offeror B's Net Present Value: ($ _____)

- Offeror ___ requires more cash outflow than Offeror ___
- Therefore, if all other terms are essentially equal, Offeror ___ is the best offer!

Discussion

Discussion. In reviewing Offeror B's purchase strategy, how would you handle an additional "tune-up" maintenance cost of $2,500 after the second year (in the 25th month)?

Class Exercise
NPV of Offeror B (with maintenance costs)

Timeframe t		Cash Flow CF	Discount Rate i	Discount Factor DF (per t and i)	PV CF X DF
Award/purchase payment					
Maintenance costs					
Sale after final year					
TOTAL NPV					

Class Exercise
NPV of each offer with Maintenance Cost?

Offeror A's Net Present Value: _____
Offeror B's Net Present Value: _____

- Offeror __ requires more cash outflow than Offeror __
- Therefore, if all other terms are essentially equal, Offeror ___ is the best offer!

What if both offers included this maintenance cash outflow at the end of the second year?

What if both offers required your customer to build a $3,500 covered parking place for the vehicle before contract award?

What Decisions Will NPV Help You Make?

- NPV Analysis can help you:
  - Decide whether it is in the best interest of the Govt. to lease or purchase
  - Decide which offer provides the best value
  - Get you to an apples-to-apples comparison of price in terms of time or 'CONSTANT' dollars
- NPV Analysis does not help you:
  - Decide whether a price is fair and reasonable
  - Assess cost realism

When am I Required to do NPV Analysis?

- OMB Circular A-94 requires NPV analysis for a lease-purchase decision when…..
  - $1M fair market value of capital asset AND
  - Capital asset is…..
    - Leased for 3 or more years OR
    - New, has economic life of < 3 years and is leased to the Govt. for 75% or more of its economic life OR
    - Built expressly for the Govt. OR
    - Leased to the Govt. and clearly no alternative commercial use

Net Present Value Analysis
Identifying Issues and Concerns

- ☑ Is net present value analysis used when appropriate?
- ☑ Are the dollar estimates for expenditures and receipts reasonable?
- ☑ Are the times projected for expenditures and receipts reasonable?
- ☑ Are the proper discount rates used in the net present value calculations?
- ☑ Are the proper discount factors used in analysis?
- ☑ Are the discount factors properly calculated from the discount rate?
- ☑ Are there any differences in the goods or services proposed? If so, what is the value of the differences.

Conclusion

You understand NPV if you understand:

- The "Time Value of Money" concept
- How the term "present value" differs from the term "net present value"
- When to use the nominal versus the real discount rate
- The difference between discount rate and discount factor
- Where to find the latest OMB discount rates
- The relationship between present value and discount rate
- How to calculate net present value of future payments (outflows) or paybacks (inflows)
- Common mistakes made in calculating NPV
- How to evaluate "lease vs. buy" alternatives

Additional NPV Practice

- Solve the following problem on your own to check for understanding
- This is not an in-class assignment
- Solution is at the back of the lesson.

Additional NPV Practice

Two offerors proposed to meet your agency's test equipment requirements with the following proposals.

- Offeror A proposes a 5-year equipment lease, which includes all maintenance costs, for a total price of $120,000. The lease's payment schedule is $24,000 upon contract award, with 4 subsequent payments of $24,000 due at the beginning of each subsequent year.
- Offeror B's proposal is for your agency to purchase the equipment for $110,000, then buy it back from you after 5 years, at a salvage value of $4,000. Under this option, your agency is also responsible for preventative maintenance and calibration at the end of year 3, for $5,500.
- Determine which is the best offer for the Government based on Net Present Value Analysis.

Additional NPV Practice

Offeror A

Timeframe t		Cash Flow CF	Discount Rate i	Discount Factor DF (per t and i)	PV CF X DF
TOTAL NPV					

Additional NPV Practice

Offeror B

Timeframe t		Cash Flow CF	Discount Rate i	Discount Factor DF (per t and i)	PV CF X DF
TOTAL NPV					

Additional NPV Practice

Offeror A's Net Present Value: ($_____)
Offeror B's Net Present Value: ($_____)

- Offeror A requires _____ cash outflow than Offeror B
- Therefore, if all other terms are essentially equal, Offeror ___ is the best offer!

Solutions:

Additional NPV Practice

Offeror A

Timeframe t		Cash Flow CF	Discount Rate i	Discount Factor DF (per t and i)	PV CF X DF
Award/first payment	0	($24,000)	1.1%	1.0000	($24,000)
Next payment	1	($24,000)	1.1%	.9891	($23,738.40)
Next payment	2	($24,000)	1.1%	.9784	($23,481.60)
Next payment	3	($24,000)	1.1%	.9677	($23,224.80)
Final payment	4	($24,000)	1.1%	.9572	($22,972.80)
TOTAL NPV					($117,417.60)

Additional NPV Practice

Offeror B

Timeframe t		Cash Flow CF	Discount Rate i	Discount Factor DF (per t and i)	PV CF X DF
Award/ Purchase payment	0	($110,000)	1.1%	1.0000	($110,000)
	1				
	2				
Maintenance	3	($5,500)	1.1%	.9677	($5,322.35)
	4				
Salvage	5	$4,000	1.1%	.9468	$3,787.20
TOTAL NPV					($111,535.15)

Additional NPV Practice

Offeror A's Net Present Value: ($117,417.60)
Offeror B's Net Present Value: ($111,535.15)

- Offeror A requires more cash outflow than Offeror B
- Therefore, if all other terms are essentially equal, Offeror B is the best offer!

Lesson 5 -- ELO 2.04 Given Market Research data calculate and identify reasonable Cost Estimating Relationships.

Cost Estimating Relationship (CER) Defined

The CPRG, Volume 2, Chapter 4, Paragraph 4.0 defines a Cost Estimating Relationship as a tool used to estimate a future cost or price by relating an independent variable to a dependent variable. Some CERs are simple ratios, such as comparing the ratio of "dollars per square foot" on a previous painting project to estimate the price of a future one. These simple ratios are often called "rules of thumb," for they become common methods for estimating prices. Other CERs are more complex, such as linear or curvilinear equations which predict the relationship between a production quantity and total price. This lesson explores how fundamental CER ratios can help us estimate future prices. The last two lessons of Unit 2 explore the use of linear equations as CERs to help us estimate future prices.

What does the FAR say about "CERs"?

As stated in the excerpt from FAR 15.404-1(b), price analysts may use parametric estimating methods as tools for price analysis for both commercial and noncommercial items.

> *(2) The Government may use various price analysis techniques and procedures to ensure a fair and reasonable price. Examples of such techniques include, but are not limited to the following:*
>
> *(iii) Use of parametric estimating methods/application of rough yardsticks (such as dollars per pound or per horsepower, or other units) to highlight significant inconsistencies that warrant additional pricing inquiry.*

Cost Estimating Relationships (CERs) are a type of parametric estimating method, which are used to estimate prices of future purchases, based upon a comparison of prices or key parameters from historical purchases. To introduce CERs, consider the following example.

Example of developing and using a CER. Assume the Navy intended to upgrade the software in a destroyer fleets' fire control system, and that this type of upgrade had not been done before on a destroyer. With no historical data on software upgrades for destroyers, the Navy team would compare fire control system software upgrades on similar ships, such as cruisers. By studying software upgrades for cruisers, the Navy team would realize "lines of code" (LOC) is a critical price driver, or "independent variable," for software systems on cruisers. Although destroyers are different than cruisers, the Navy team could still establish a basis for estimating the price of the destroyer system upgrade by establishing a CER with the cruiser system. With the data below, how could you estimate the price of the destroyer's software upgrade?

4 years ago	Cruiser System AA	800,000 LOC	$8,000,000
2 years ago	Cruiser System BB	1,000,000 LOC	$9,500,000
Today	Destroyer System DD	1,200,000 LOC (estimate)	$ ___**?**___

Discussion.

- By this CER, or "rule of thumb" for estimating software upgrades on Navy cruisers, what would we estimate the price of an upgrade on a destroyer with roughly 750,000 lines of code?

- What other factors would you consider to build confidence in this price estimate?

- What if we estimated our destroyer project would be 3 million LOC?

- How much do we know about the historical acquisitions, such as:
  - Are we certain the historical price was fair and reasonable?

  - Did the historical acquisition face urgent timelines, many changes, surged operations?

  - Do we have an adequate number of historical data points?

This example, as well as the CPRG reference reveal that pricing with CERs requires us to know the independent variables for every requirement. Independent variables are typically the cost or performance "drivers," which significantly influence the performance and/or price of the

procurement. Consider the following categories of products or services, identify the independent variables, then state examples of possible CERs. The first one is done as an example.

Product or Service	**Independent Variable**	**Possible CERs**
Construction	Floor space, square footage, surface area, wall surface	$ per square foot, $ per floor
Roofing		
Shipping/Delivery		
Reciprocating Engines (gasoline, automobile, tractor, generator)		
Turbine Engines		
Aircraft		
Sheet metal		
Security service		

Discussion.

- How can we estimate future prices for security service if no two contracts are exactly the same?

- How can we estimate future prices for construction projects if we can not find two construction projects exactly the same?

Steps for Developing a Cost Estimating Relationship

As we saw from the previous example, there can be several possible CERs for estimating a future price of products and services. Now we will learn how to pick the best CER. Strictly speaking, a CER is not a quantitative technique. Rather, CERs are a framework for using appropriate quantitative techniques to quantify a relationship between an independent variable and contract cost or price. In a sense, CERs are relationships we can use to compare prices of *similar* items, then estimate prices of similar items when it is not possible to compare the *same* items. In developing CERs, the CPRG recommends the following six step process:

Step 1. Define the dependent variable (e.g., cost dollars, hours, and so forth) Define what the CER will estimate. Will the CER be used to estimate price, cost dollars, labor hours, material cost, or some other measure of cost? Will the CER be used to estimate total product cost or estimate the cost of one or more components? The better the definition of the dependent variable, the easier it will be to gather comparable data for CER development.

Step 2. Select independent variables to be tested for developing estimates of the dependent variable. In selecting potential independent variables for CER development:

- Draw on personnel experience, the experience of others, and published sources of information. When developing a CER for a new state-of-the-art item, consult experts experienced with the appropriate technology and production methods.

- Consider the following factors:

  - Variables should be quantitatively measurable. Parameters such as maintainability are difficult to use in estimating because they are difficult to measure quantitatively.

  - Data availability is also important. If you cannot obtain historical data, it will be impossible to analyze and use the variable as a predictive tool. For example, an independent variable such as physical dimensions or parts count would be of little value during the conceptual phase of system development when the values of the independent variables are not known. Be especially wary of any CER based on 2 or 3 data observations.

  - If there is a choice between developing a CER based on performance or physical characteristics, performance characteristics are generally the better choice, because performance characteristics are usually known before design characteristics.

Step 3. Collect data concerning the relationship between the dependent and independent variables. Collecting data is usually the most difficult and time-consuming element of CER development. It is essential that all data be checked and double checked to ensure that all observations are relevant, comparable, relatively free of unusual costs.

Step 4. Explore the relationship between the dependent and independent variables. During this step, you must determine the strength of the relationship between the independent and dependent variables. This phase of CER development can involve a variety of analytical techniques from simple graphic analysis to complex mathematical analysis. Simple ratio analysis, moving averages, and linear regression are some of the more commonly used quantitative techniques used in analysis.

Step 5. Select the relationship that best predicts the dependent variable. After exploring a variety of CERs, select the CER with the least amount of variation among several data points between the independent and dependent variables.

Step 6. Document your findings. CER documentation is essential to permit others involved in the estimating process to trace the steps involved in developing the relationship. Documentation should involve the independent variables tested, the data gathered, sources of data, time period of the data, and any adjustments made to the data.

Now, considering this lesson, and the CPRG Volume 2, Chapter 4, Sections 4.0 through 4.4, consider two examples:

Example 1. Which of the following CERs would be best to use to estimate a reasonable price for buying a residential house: $ per Bedroom, $ per Acre, or $ per Square Foot?

Step 1. Define the dependent variable.

Step 2. Select independent variables to be tested for developing estimates of the dependent variable. *(In this problem, based on Market Research, the independent variables are given: $/Bedroom, $/Acre, and $/Sq Ft).*

Step 3. Collect data concerning the relationship between the dependent and independent variables. *See following table of Market Research Data.*

Market Research Data:

House	Price	#BR	$ / BR	Acres	$ / Acre	Sq ft	$ / Sq ft
1	$350K	4		.5		3500	
2	$297K	3		2		3000	
3	$409K	4		.5		3900	
4	$307K	3		1		3200	
5	$303K	4		5		3000	
6	$306K	3		.5		3100	
Range							
Avg							
Range/ Avg							

Step 4. Explore the relationship between the dependent and independent variables.

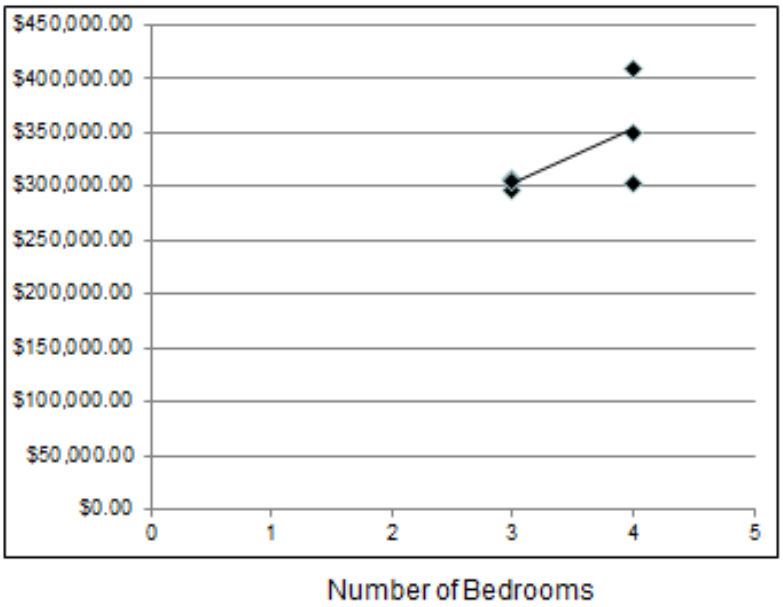
Step 5. Select the relationship that best predicts the dependent variable.

Step 6. Document your findings.

Student note. The following graphs depict another method for determining the variance in a CER. Analyzing these graphs is optional in CON 170, but provides insight into tools and analysis to be learned in CON 270. The graphs plot the independent variable on the x-axis (number of bedrooms, number of acres, and number of square feet), and the dependent variable (price) on the y-axis. The graph also includes the "trendline," which is the line that represents the best fit of the general trend of the data points. Looking at the graphs, which appears to have the most linear trend, where all the data points fall on or near a straight line? The "$ per square feet" line presents the most linear trend. Thus, we would expect "$ per square feet" to yield the most accurate prediction of future prices of residential homes.

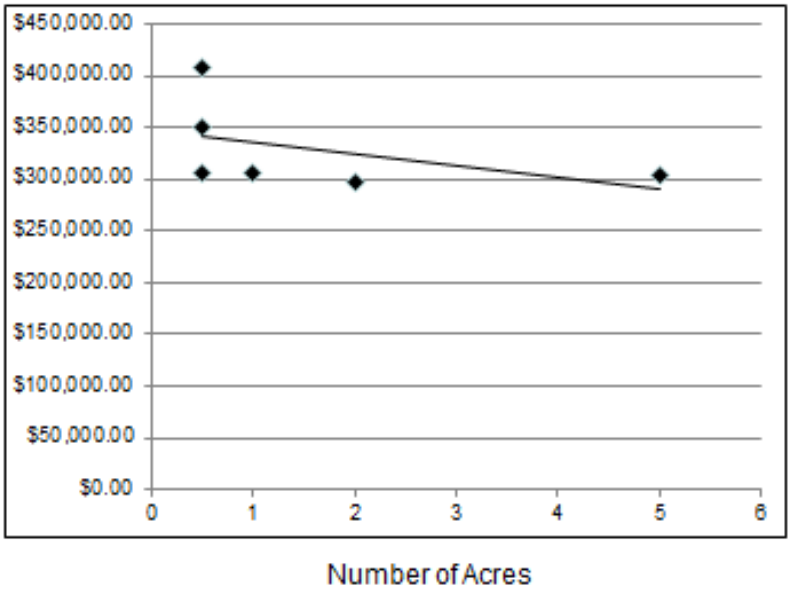
Residential House Prices
Price Per Bedroom

Bedrooms	Price
4	$350,000.00
3	$297,000.00
4	$409,000.00
4	$307,000.00
4	$303,000.00
3	$306,000.00



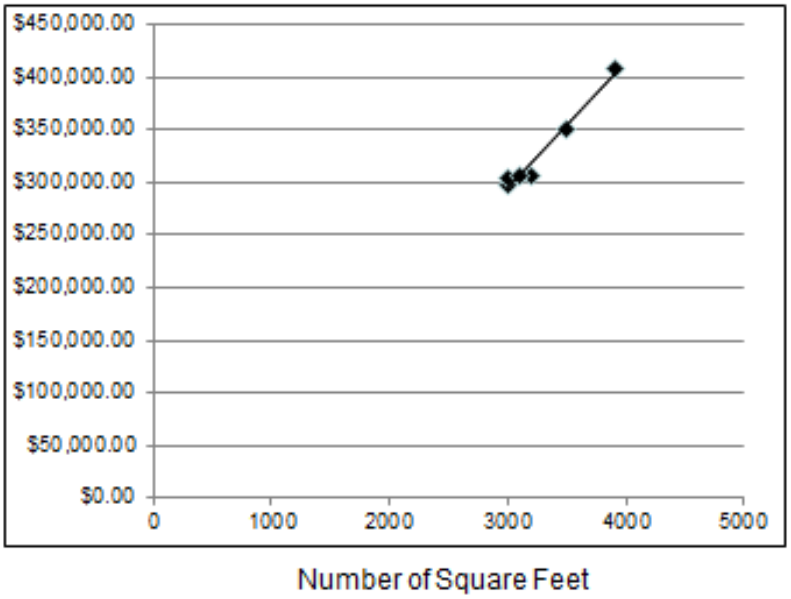
Residential House Prices
Price Per Acre

Acres	Price
0.5	$350,000.00
2	$297,000.00
0.5	$380,000.00
1	$307,000.00
5	$303,000.00
0.5	$306,000.00



Residential House Prices
Price Per Square Foot

Square Feet	Price
3500	$350,000.00
3000	$297,000.00
3900	$409,000.00
3200	$307,000.00
3000	$303,000.00
3100	$306,000.00



Example 2. Given the following data regarding Security Guard Services, what would be a reasonable CER for buying security guard services?

Step 1. Define the dependent variable.

Step 2. Select independent variables to be tested for developing estimates of the dependent variable.

Step 3. Collect data concerning the relationship between the dependent and independent variables. *See following table of Market Research Data.*

Market Research Data:

Contract	**Price**	**Gates**	**$ per Gate**	**Hours**	**$ per hr**
1	$2,080,000	6		208,000	
2	$1,575,000	1		150,000	
3	$3,800,000	10		400,000	
Range					
Avg					
Range/Avg					

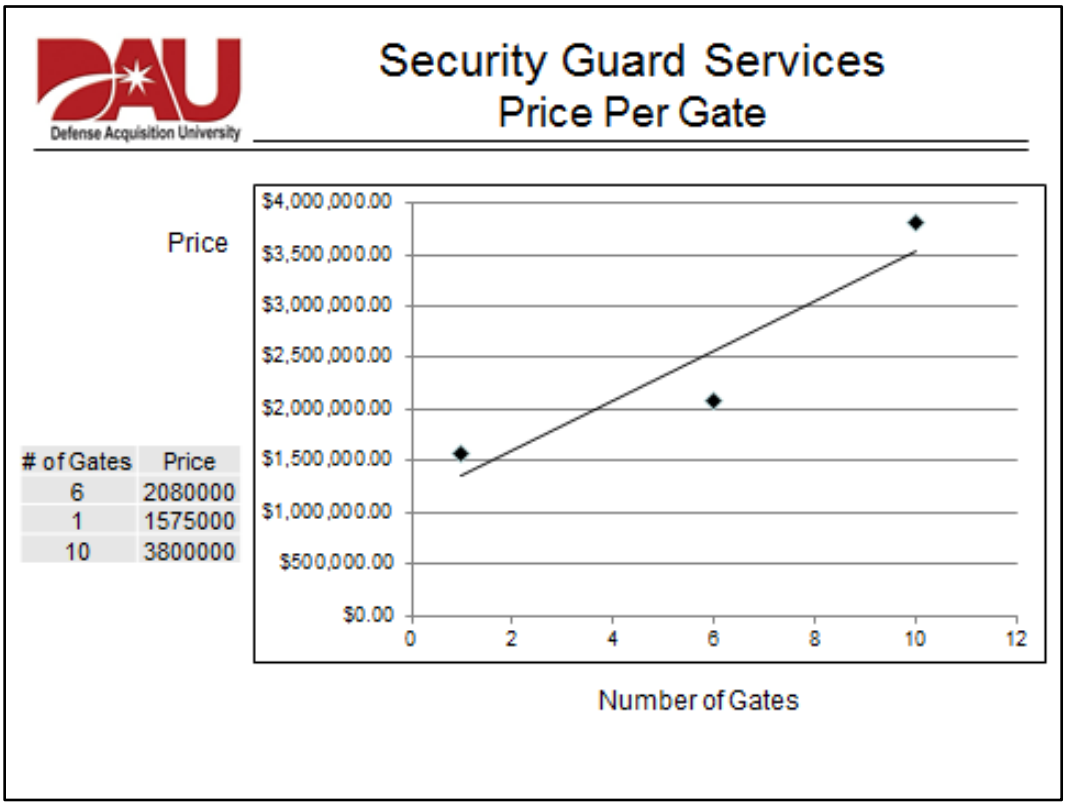
Step 4. Explore the relationship between the dependent and independent variables.

Step 5. Select the CER that best predicts the dependent variable.

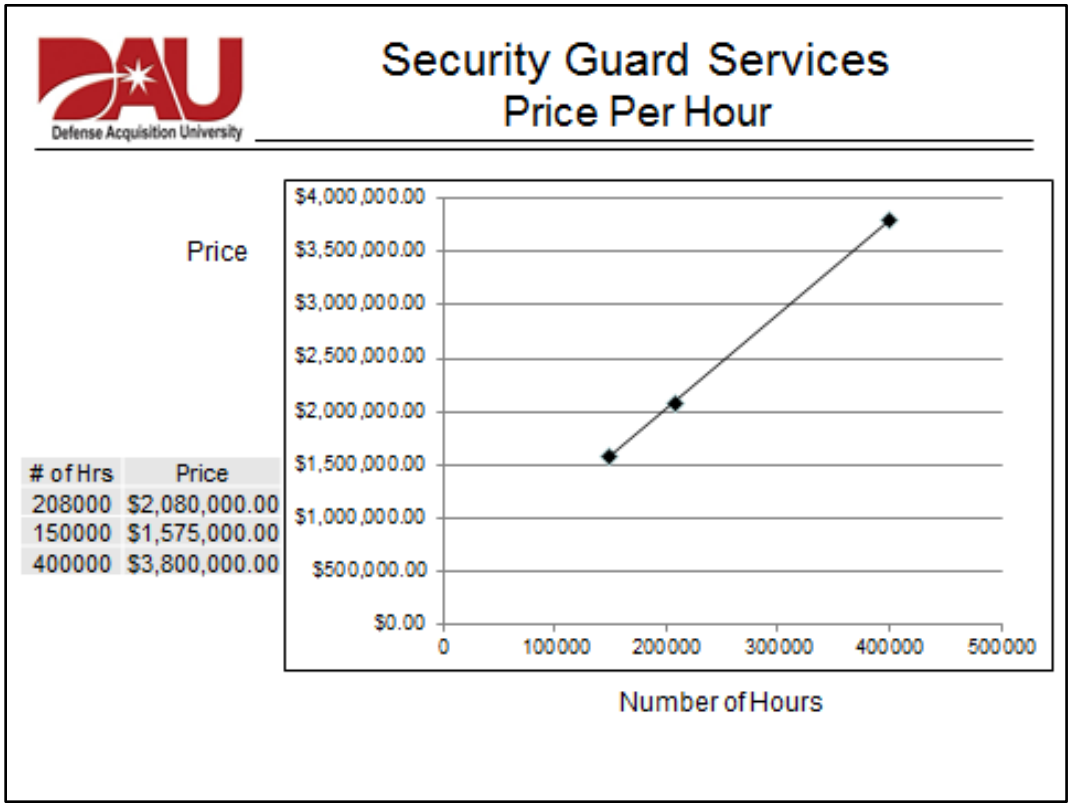
Step 6. Document your findings.

Student note. As with Example 1, the following graphs depict another method for determining the variance in a CER. Analyzing these graphs is optional in CON 170, but provides insight into tools and analysis to be learned in CON 270, using Microsoft Excel. The graphs plot the independent variable on the x-axis ($/Gates and $/hour), with the dependent variable (price) on the y-axis. The graph also includes the "trendline," which is the line that represents the best fit of the general trend of the data points. Which of the two graphs appears to present the most linear trend, where all the data points fall on or near a straight line? The "$ per hour" line presents the most linear trend. Thus,

we would expect "$ per hour" to yield the most accurate prediction of future prices for security services.



# of Gates	Price
6	2080000
1	1575000
10	3800000



# of Hrs	Price
208000	$2,080,000.00
150000	$1,575,000.00
400000	$3,800,000.00

Advantages to Using CERs

Versatility. If the data are available, parametric relationships can be derived at any level, whether system or subsystem component. As the design changes, CERs can be quickly modified and used to answer what-if questions about design alternatives.

Sensitivity. Simply varying input parameters and recording the resulting changes in cost can produce a sensitivity analysis.

Statistical output. Parametric relationships derived from statistical analysis generally have both objective measures of validity (statistical significance of each estimated coefficient and of the model as a whole) and a calculated standard error that can be used in risk analysis. This information can be used to provide a confidence level for the estimate, based on the CER's predictive capability.

Objectivity. CERs rely on historical data that provide objective results. This increases the estimate's defensibility.

Disadvantages of Using CERs

Database requirements. The historical data and database must be consistent and reliable. It may be time consuming to normalize the data or to ensure that the data were normalized correctly (with respect to time, performance parameters, key terms of the acquisition), especially if someone outside the estimator's team developed the CER. Without understanding how the data were normalized, the analyst has to accept the database on faith—sometimes called the "black-box syndrome," in which the analyst simply plugs in numbers and unquestioningly accepts the results. Using a CER in this manner increases the estimate's risk of producing a faulty estimate.

Relevance. Using data outside the CER range may cause errors, because the CER loses its predictive ability for data outside the development range.

Complexity. Complicated CERs (such as nonlinear CERs) may make it difficult for others to readily understand the relationship between cost and its independent variables.

Additional Guidance and Points to Consider

- Some CERs may be simple, linear ratios, while others may be more complex, non-linear relationships.

- The CPRG cautions buyers to beware of data samples of only 2 or 3 data observations (Vol 2 Ch 4, Para 4.0, Step 2). In such cases, you may not have enough information to validate the usefulness of a CER, and should pursue additional data points, as well as expert advice.

- When developing CERs, do other subject matter experts agree that your independent variables are drivers?

Conclusion

After completing these exercises, we have learned to use market research data to identify and calculate Cost Estimating Relationships. We have learned to identify independent variables, which are the "drivers" of performance and price. We understand some CERs are simple ratios, which are often considered "rules of thumb," for developing price estimates. Finally, we understand how to select the best CER among several possible CERs by seeking the CER with the least amount of variation between the independent and dependent variables.

In the next two lessons, we will explore more complex CERs, which are based on linear equations.

Lesson 6 -- Cost-Volume Analysis

ELO 2.05 Through Cost-Volume Analysis, recognize the nature of fixed, variable, semi-variable and total costs, and develop a price estimate.

ELO 2.06 Through Cost-Volume Analysis, determine a proposed price to be rational or irrational with respect to a "buy-in" seller strategy.

Introduction, Background, and Assumptions

When you acquire supplies or services, you expect to pay a lower price per unit as the purchase quantity increases. You expect contractors to have lower costs per unit as production quantity increases. This general expectation remains the same whether you are buying items specifically built for the Government, or items that are mass-produced for a variety of commercial and Government customers. Cost-volume analysis can be used to analyze the natural relationship between cost and volume in pricing decisions.

Assumptions. In cost-volume analysis, you consider only short-term operations. The short term may be defined as a period too short to permit facilities expansion or contraction, or other changes that might affect overall pricing relationships.

Linear Relationship. The CVA technique also assumes use of a straight line relationship (not curvilinear) between cost and volume. While actual price behavior may not follow a straight line, its use can closely approximate actual cost behavior in the short run. If purchase volume moves outside the relevant range of the available data, the straight-line assumption and the accuracy of estimates becomes questionable.

Cost and Price. Finally, the CVA technique assumes the terms "cost" and "price" are the same. The "cost" data points in CVA could refer to either "prices" discovered through market research, or could refer to actual "costs" to produce the good or service. The important point is to avoid mixing "cost" and "price" data points into the same data set. In CON 170, the data points used in the CVA examples refer to prices paid for goods or services.

Let's consider the following example as we begin our exploration of Cost-Volume Analysis.

Example 1. Wiring Harnesses. Given market research data for commercial grade wiring harnesses, estimate the price for an upcoming purchase of 18 harnesses.

Data Set 1: Commercial Grade Wiring Harnesses

Quantity	Total Cost	Unit Cost
5	$150.00	$30.00
10	$200.00	$20.00
15	$250.00	$16.67
20	$300.00	$15.00
8	$180.00	$22.50
12	$220.00	$18.33
17	$270.00	$15.88
25	$350.00	$14.00

Step 1: To begin, plot the data provided on graph paper.

Step 2: Establish a "trendline" or estimating line by connecting the data points

Step 3: Based on the graph and estimating line, what is your estimate for the price of 18?

Discussion.

- From the graph, notice the estimating line slopes up to the right. This means for every additional unit produced, the total cost increases. This is called a positive slope—it moves up and to the right with each additional unit purchased. The increase in total cost

with the purchase of an additional unit is the variable cost. Thus, the "slope" of the estimating line is the variable cost.

- From this data set and estimating line, could you estimate the price of 50 harnesses? 200?

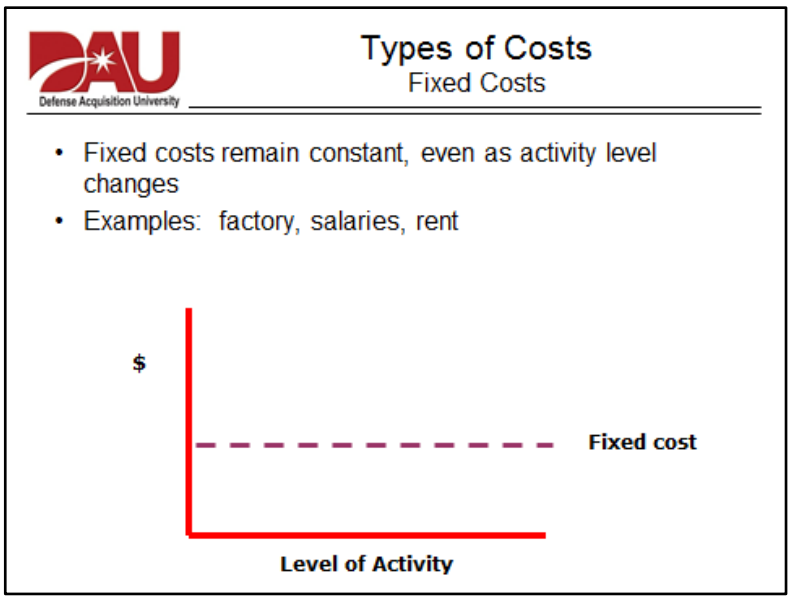
Can we sharpen our cost estimate? Having the graph is helpful, but with the tools from Lesson 1, we can determine the mathematical expression for the estimating line we drew in the first part of this lesson. This process of analyzing the relationship between cost and volume is known as "Cost-Volume Analysis," or "CVA." CVA is a more complex form of Cost Estimating Relationships than the simple ratios we learned in our last lesson. Before we explore the use of CVA, we must first understand the types of costs involved.

Types of Costs

In the short run, costs can be of three general types:

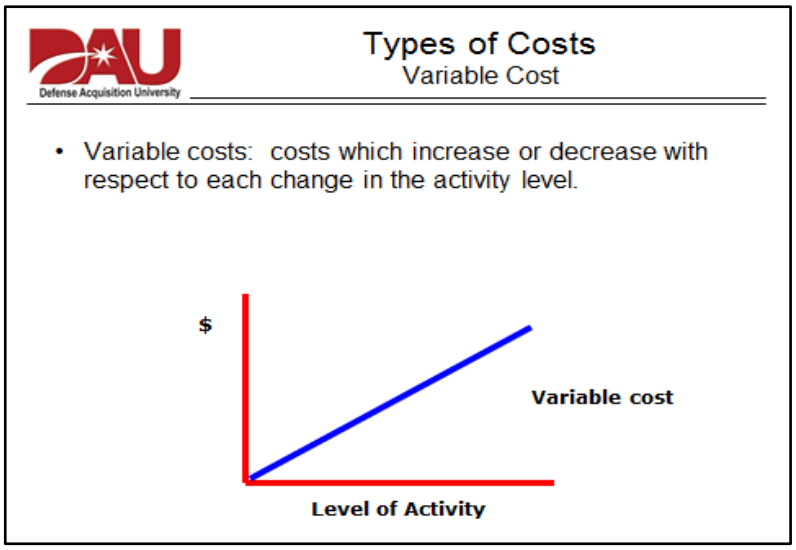
Fixed Cost (symbol = F). *Total* fixed costs remain constant as volume varies in the relevant range of production. Fixed cost *per unit* decreases as the cost is spread over an increasing number of units.

Examples include: Fire insurance, depreciation, facility rent, and property taxes.



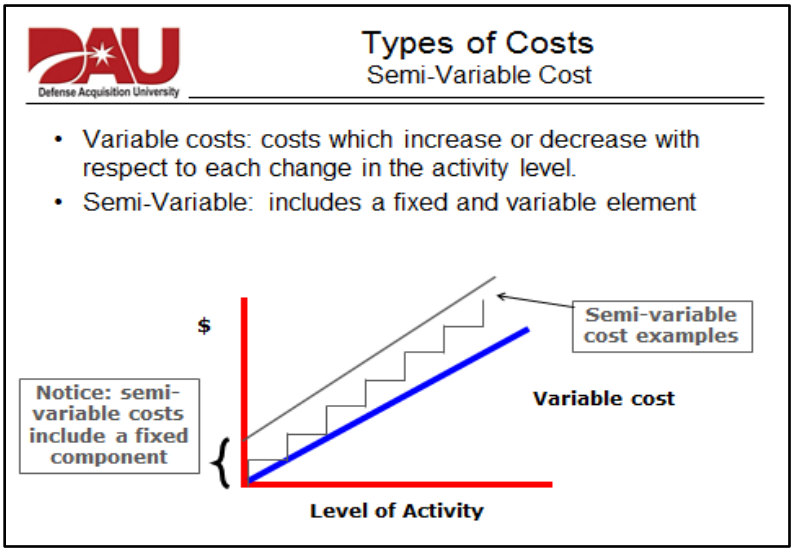
Variable Cost (symbol = Vu). Variable cost *per unit* remains constant no matter how many units are made in the relevant range of production. *Total* variable cost increases as the number of units increases.

Examples include: Production material and labor. If no units are made, neither cost is necessary or incurred. However, each unit produced requires production material and labor.



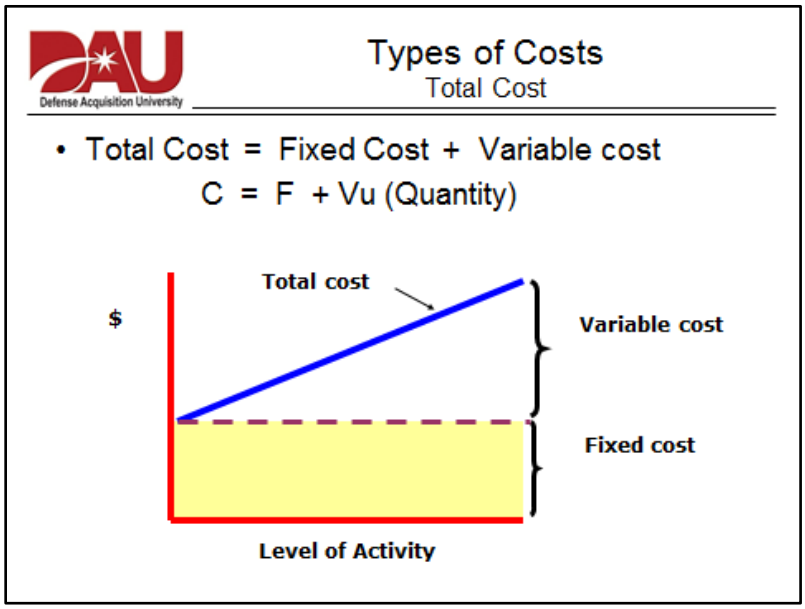
Semi-variable Cost (symbol = SVu). Semi-variable costs include both fixed and variable cost elements. These kinds of costs may increase in steps or increase relatively smoothly from a fixed base.

Examples include: Supervision and utilities, such as electricity, gas, and telephone. Supervision costs tend to increase in steps as a supervisor's span of control is reached. Utilities typically have a minimum service fee, with costs increasing relatively smoothly as more of the utility is used.



Total Cost. Fixed, Variable and Semi-Variable Costs, when added together will give us the **Total Cost (symbol = C)** which is the sum of fixed and variable costs.

Examples include: the total cost for production includes the cost to build the factory (fixed cost), plus the cost of utilities and labor (variable costs). Total cost can be considered a "semi-variable" cost because it contains both fixed and variable cost elements.



Situations for Using CVA

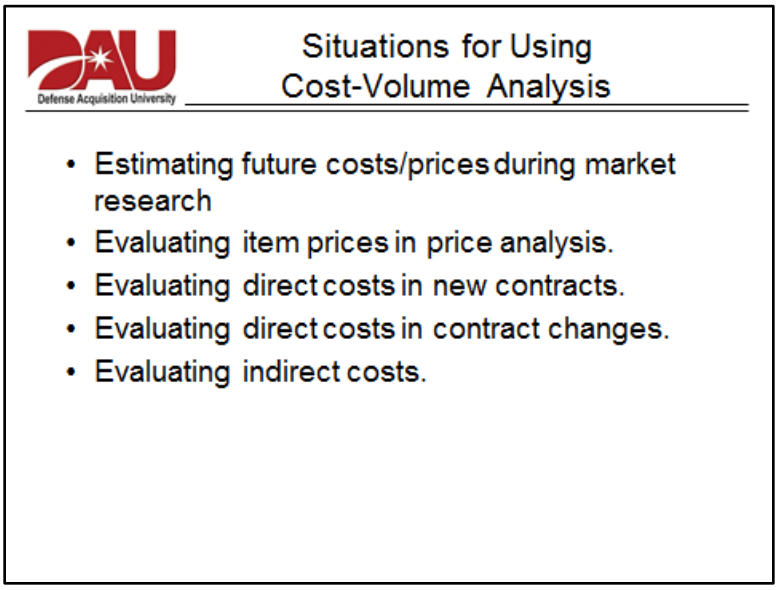
Cost-volume analysis is an estimating concept that can be used in a variety of pricing situations. You can use the cost-volume relationship for:

Evaluating item price in price analysis. Cost-volume analysis assumes that total cost is composed of fixed and variable elements. This assumption can be used to explain price changes as well as cost changes. As the volume being acquired increases unit costs decline. As unit costs decline, the vendor can reduce prices and still make the same profit per unit.

Evaluating direct costs in pricing new contracts. Quantity differences will often affect direct costs -- particularly direct material cost. Direct material requirements often include a fixed component for development or production operation set-up. As that direct cost is spread over an increasing volume unit costs should decline.

Evaluating direct costs in pricing contract changes. How will an increase in contract effort increase contract price? Some costs will increase others will not. The concepts of cost-volume analysis can be an invaluable aid in considering the effect of the change on contract price.

Evaluating indirect costs. The principles of cost-volume analysis can be used in indirect cost analysis. Many indirect costs are fixed or semi-variable. As overall volume increases, indirect cost rates decline because fixed costs are spread over an increasing production volume.



Analyzing the Cost-Volume Relationship

The assumption of linear cost behavior permits use of straight-line graphs and simple linear algebra in cost-volume analysis. This lesson began with the graphical analysis, and then presents the algebraic analysis.

Calculating the Total Cost. Total cost is a semi-variable cost—some costs are fixed, some costs are variable, and others are semi-variable. In analysis, the fixed component of a semi-variable cost can be treated like any other fixed cost. The variable component can be treated like any other variable cost. As a result, we can say that:

Total Cost = Fixed Cost + Variable Cost

Using symbols: C = F + V, where

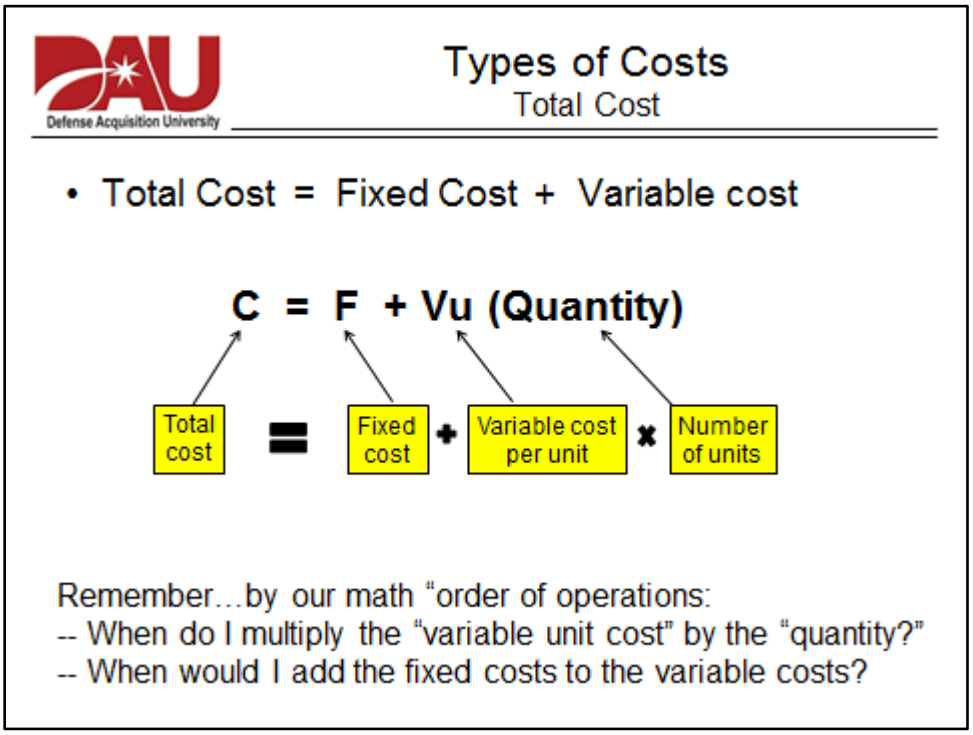
- C = Total Cost;
- F = Fixed Cost;
- V = Variable Cost

Variable Cost, represented by "V," is a product of two elements:

- Variable Cost per Unit, Vu
- and Quantity Produced, Q

Substituting this information into the Total Cost equation, we have the equation used in cost-volume analysis:

$$\mathbf{C = F + V_U\,(Q)}$$



Example 2. For a certain widget, Fixed Costs are $500, Variable Cost per Unit is $10, and the Volume produced is 1,000 units. The Total Cost equation enables us to calculate the Total Cost of production.

$C = F + V_U(Q)$

$C = \$500 + \$10\ (1{,}000)$

$C = \$500 + \$10{,}000$

$C = \$10{,}500$

If our Total Cost of 1,000 widgets is $10,500, what is our Total Cost per Unit? (Note, this is different than the Variable Cost per Unit, Vu)

Total Cost per Unit = Total Cost / Number of units

= $10,500 / 1,000 units

= $10.50 per unit

Additional Examples of Calculating Total Cost:

Cost-Volume Analysis

If you were trying to estimate the cost of purchasing 1,000 "wiz-bangs" that had a FIXED cost of operations of $5,000 and a VARIABLE cost per unit of $20, what would you estimate your total cost to be?

From what we learned: $C = F + V_U(Q)$

Cost-Volume Analysis

If you were trying to estimate the cost of purchasing 2,000 "wiz-bangs" that had a FIXED cost of operations of $5,000 and a VARIABLE cost per unit of $20, what would you estimate your total cost to be?

$$C = F + V_U(Q)$$

Cost-Volume Analysis

Compare: the unit cost at 2,000 units of production is less than the unit cost at 1,000 units of production, based on same equation and production line.

Why? Fixed costs are shared, or spread across more units!

$$C = F + V_U(Q)$$

At 1,000 units: At 2,000 units:

Develop a Cost Estimate with CVA

There are 4 steps to estimating the cost for a given quantity using historical data:

Step 1. Calculate the variable element.
Step 2. Calculate the fixed element.
Step 3. Develop the Estimating Equation.
Step 4. Estimate the cost for a given quantity.

Example 3. The example below illustrates how to use CVA to estimate the price for a certain quantity of items.

Using Cost-Volume Analysis
Example 3

Given historical data, estimate the price you would expect to pay for 4,400 wiz-bangs.

Historical Data

Cost	Qty
$40,000	3000
$50,000	4000
$60,000	5000

4,400

Step 1. Calculate the variable element - Given Total Cost and Volume for two different levels of production, and the straight-line assumption, you can calculate Variable Cost per Unit.

Remember:

1. Fixed Costs do NOT change as volume changes, assuming we remain within the relevant range of production. Thus, a change in Total Cost is the result of a change in Variable Cost (which is Vu(Q)).
2. Variable Cost per Unit does NOT change in the relevant range of production.

As a result, we can calculate Variable Cost per unit (V_U) by:

$$V_u = \frac{\text{Change in Total Cost}}{\text{Change in Quantity}}$$

$$V_u = \frac{\text{Total Cost at Point 2 - Total Cost at Point 1}}{\text{Quantity at Point 2 - Quantity at Point 1}}$$

$$V_u = \frac{C2 - C1}{Q2 - Q1}$$

1. Calculate the Variable Element

- Given historical price data:

Cost	Qty
$40,000	3000
$50,000	4000
$60,000	5000

- Calculate the variable cost element like this:
  - Pick two cost/quantity points
  - Use the Variable Cost/unit equation: $V_U = \frac{C_2 - C_1}{Q_2 - Q_1}$
- Do the math:

$$V_U = \frac{60{,}000 - 40{,}000}{5{,}000 - 3{,}000}$$

$$V_U = \frac{20{,}000}{2{,}000}$$

$$V_U = \$10$$

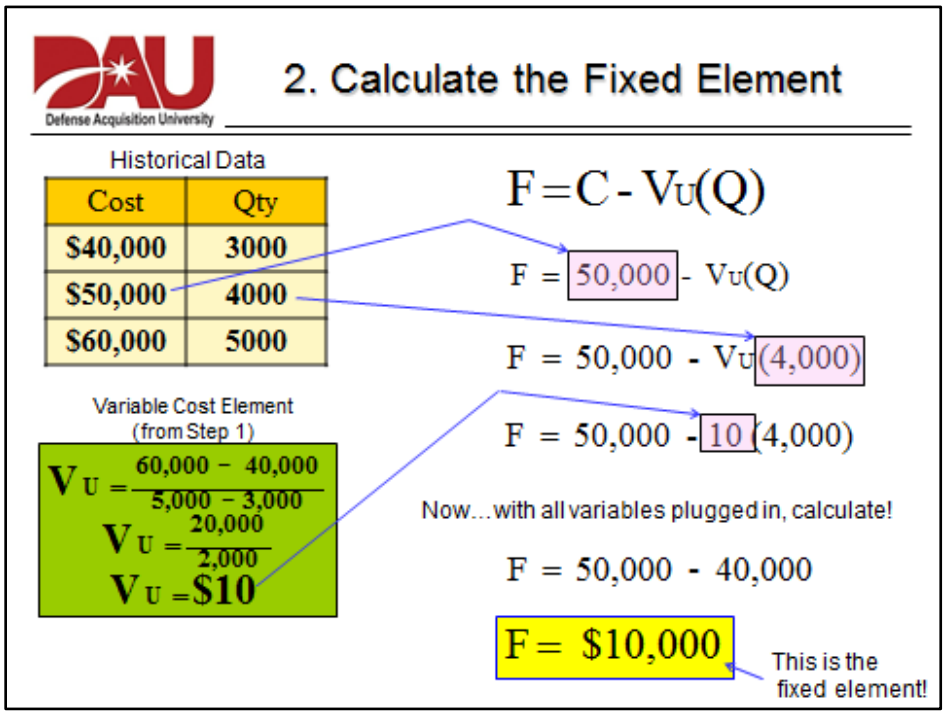
$10 is the "variable cost per unit," or "variable element"

Step 2. Calculate the fixed element - If you know Total Cost and Variable Cost per Unit for any Quantity, you can calculate Fixed Cost using the basic Total Cost equation.

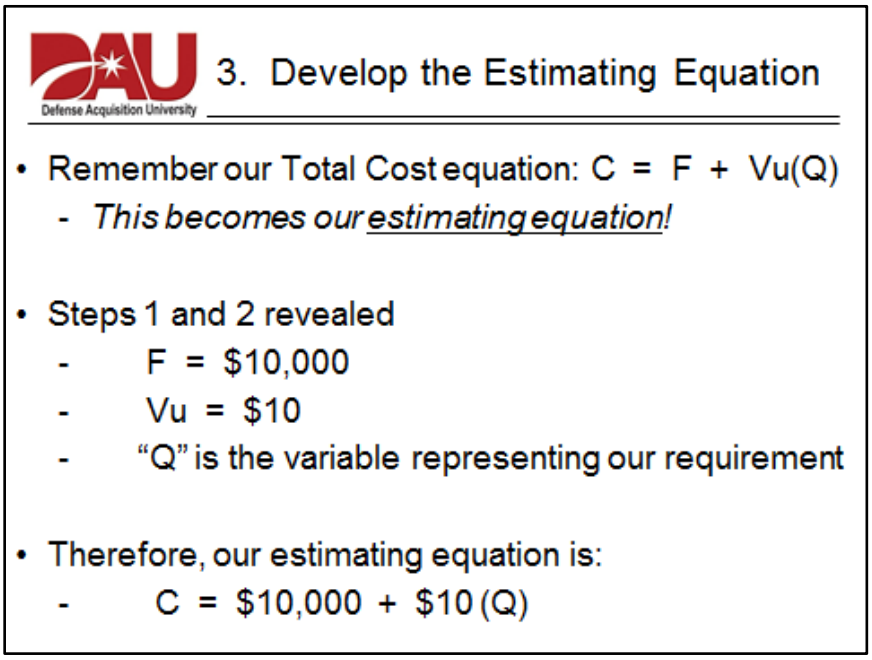
2. Calculate the Fixed Element

- Remember our Total Cost equation: $C = F + V_U(Q)$
- To calculate the Fixed Element, manipulate equation: $F = C - V_U(Q)$
- Given the same historical data, plus "Vu" from Step 1, calculate the Fixed Price element:

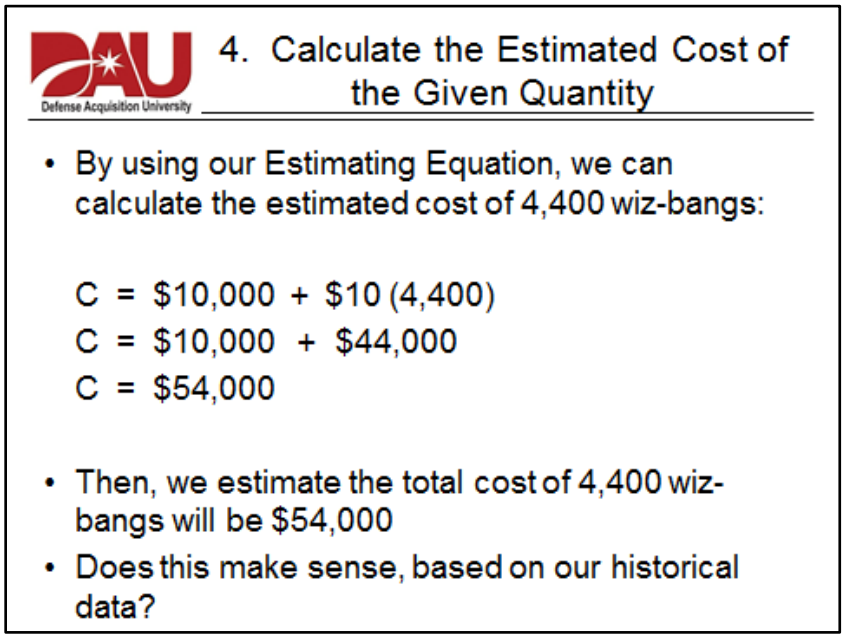
(see next slide)



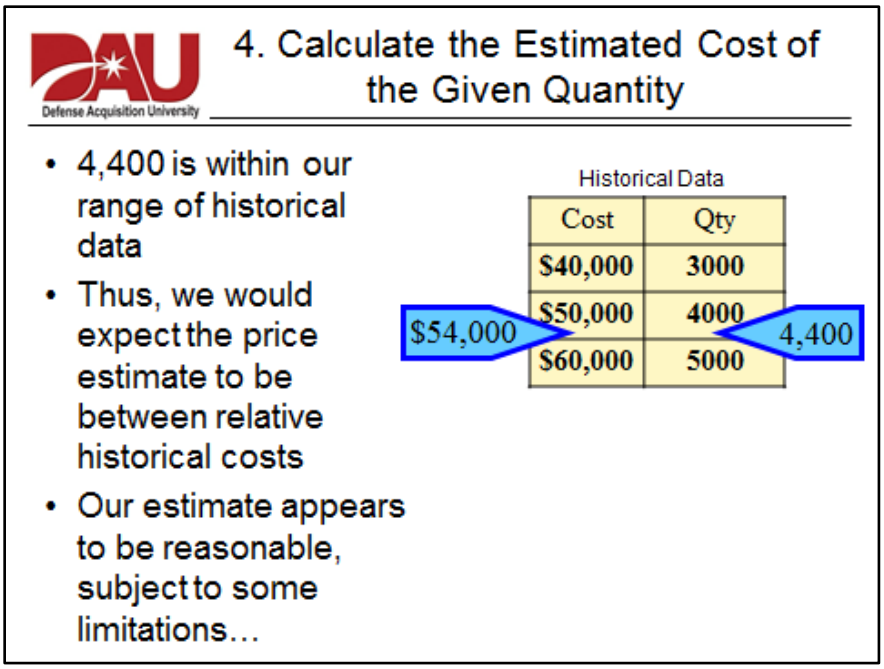
Step 3. Develop the Estimating Equation. Now that we know Vu is $10 and F is $10,000, we can substitute the values into the general Total Cost Equation. The result is the "estimating equation," which can be used to estimate the total cost of any volume in (or at least near) our relevant, historical range of 4,000 and 5,000 units.



Step 4. Estimate the cost for a given quantity. Our task in this example is to estimate the price for a quantity of 4,400 units. Now that we have an estimating equation, we can complete our task.



Student note: the last question on the slide asks, "does this estimate of $54,000 make sense." The following slide indicates $54,000 is roughly the same relative distance between $50,000 and $60,000 as the 4,400 quantity is between the 4,000 and 5,000 quantity. Therefore, the estimate appears to be within a reasonable range.



Student note: the slide above concludes with "subject to some limitations…" Such limitations are included below.

Cost-Volume Analysis Limitations

- Great estimating tool, but also consider:
  - How old is our historical data
  - Changed market conditions (demand, competition)
  - Order sizes, location, delivery terms/timelines
  - Quality, warranty terms
  - Business strategies driving changes in supply
  - Straight "total cost line," or learning curve influence
- Market research is the key to factoring these elements in to the estimate

Example 4. Now, try an example using Cost-Volume Analysis on your own, using the 4-step process.

Using Cost-Volume Analysis
Example 4

Now, you try it! After finding the following market research data, estimate the price of 116 units:

$?

PRICE	QTY
$29,500	100
$37,500	150
$49,500	225

116

4 Steps to Develop a Cost Estimate with C-V Analysis

1. Calculate the Variable Element
2. Calculate the Fixed Element
3. Develop the Estimating Equation
4. Calculate the estimated cost for the given quantity

1. Based on data set, calculate the variable cost for each unit:

$$V_U = \frac{C_2 - C_1}{Q_2 - Q_1}$$

2. Using data set 1 and the variable cost, calculate the fixed cost element.

$$F = C - V_U(Q)$$

3. Based on questions 1 and 2, write the estimating line equation.

$$C = F + V_U(Q)$$

Based on this estimating equation, hypothetically, what is the cost to produce zero wiring harnesses?

4. Now, per our customer's request, calculate an estimate for the price of 116 units:

Does this estimate appear to be reasonable?

Example 5. Re-examine Example 1 from the beginning of the lesson, where you developed a price estimate for 18 wiring harnesses with a simple graph. To conclude this lesson, "sharpen" your price estimate for 18 wiring harnesses by using CVA.

Data Set 1 – Commercial Grade Wiring Harnesses

Quantity	Total Cost	Unit Cost
5	$150.00	$30.00
10	$200.00	$20.00
15	$250.00	$16.67
20	$300.00	$15.00
8	$180.00	$22.50
12	$220.00	$18.33
17	$270.00	$15.88
25	$350.00	$14.00

1. Based on data set 1, calculate the variable cost for a wiring harness:

$$V_U = \frac{C_2 - C_1}{Q_2 - Q_1}$$

2. Using data set 1 and the variable cost, calculate the fixed cost element.

$$F = C - V_U(Q)$$

3. Based on answers from questions 1 and 2, write the estimating line's equation!

$$C = F + V_U(Q)$$

4. Now, estimate what the customer asked us for…the price of 18 units:

Based on this estimating equation, hypothetically, what is the cost to produce zero wiring harnesses?

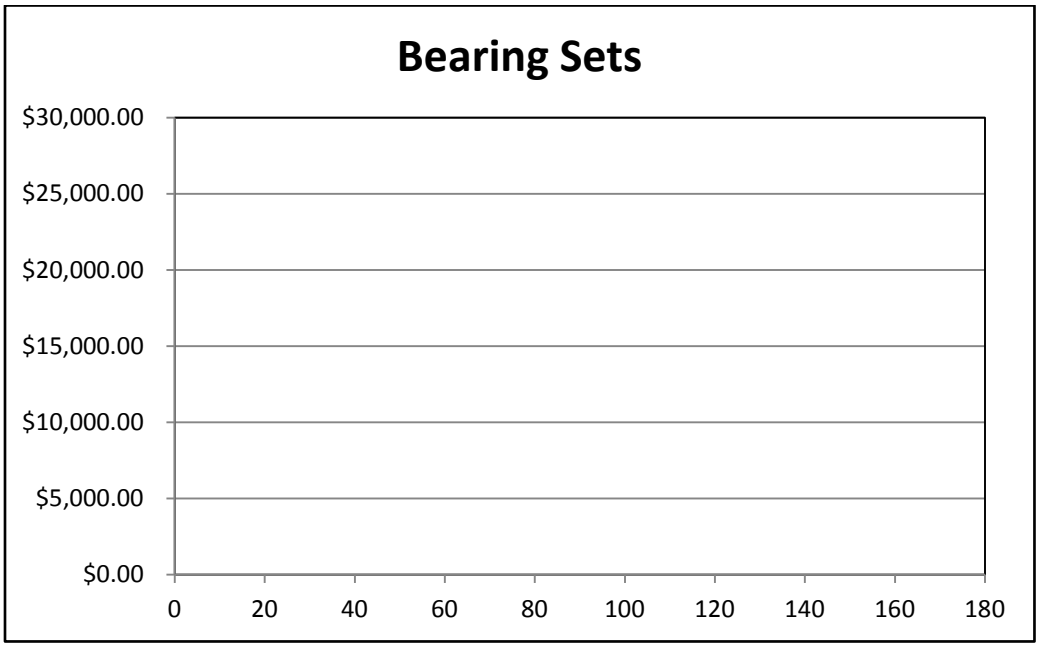
Does this appear to be close to the estimate we figured simply by looking at the graph at the beginning of this lesson?

Exercise 6. Given the market research data set below for tracked vehicle ball-bearing sets, use both graphical analysis and Cost-Volume Analysis to estimate the price for 130 ball-bearing sets.

Tracked Vehicle Ball-bearing Sets

Quantity	Total Cost	Unit Cost
20	$7,500.00	$375.00
40	$10,000.00	$250.00
60	$12,500.00	$208.33
80	$15,000.00	$187.50
100	$17,500.00	$175.00
120	$20,000.00	$166.67
140	$22,500.00	$160.71
160	$25,000.00	$156.25

1. Graph the data of Quantity in relation to Total Cost.



2. Calculate the variable cost:

$$V_U = \frac{C_2 - C_1}{Q_2 - Q_1}$$

3. Using the data set and the variable cost, calculate the fixed cost element.

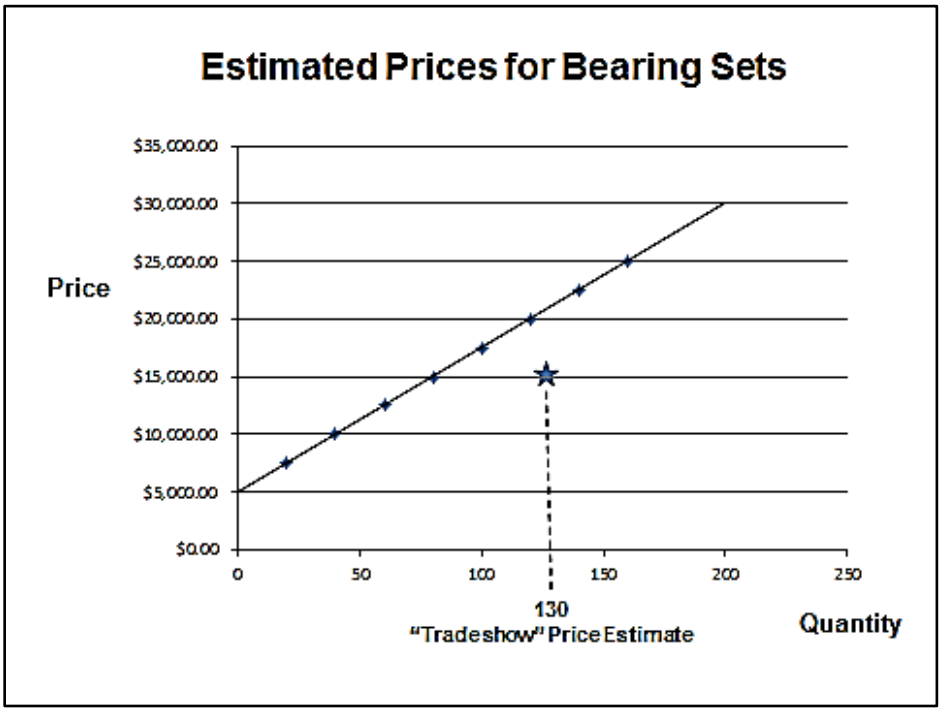
4. Based on answers to questions 1 and 2, write the estimating line's equation!

5. Based on the customer's request, what is the price estimate for 130 ball bearing sets:

6. How about the estimated price for 200 sets?

7. Based on your market research, what is the lowest price you would reasonably expect to pay?

8a. After a recent, city-wide tradeshow, a small business representative visits your office, and says he can deliver 130 units for a total cost of $16,500. On the graph below, you compared this price with the rest of your market research data. Based on your market research data, and your cost-volume analysis, do you believe this is a reasonable offer? What questions would you ask?



8b. The offeror responds to your questions, "I'm in a business cycle where I have received more orders that I expected on other contracts. Therefore, all of my fixed costs are covered for the rest of this year. For the rest of this year, I can offer you a better price than normal." What do you think?

C-V Analysis Summary

- C-V Analysis:
  - Enables us to use historical data to estimate fixed, variable and total costs
  - Enables the Gov't to estimate future prices based on historical data
  - Gives the Gov't insight regarding contractor's business strategy and production capacity
  - Is useful, but has limitations we must address

Solutions to Lesson 6, Exercises 5 and 6

Exercise 5

Wiring Harness Estimate with CVA
Example 5

1. Calculate the Variable Element

$$\frac{\$200-\$150}{10-5}=\boxed{\$10}$$

2. Calculate the Fixed Element

$$Fixed=\$150-\$10(5)=\boxed{\$100}$$

3. Develop the Estimating Equation

$$Cost=\boxed{\$100+\$10(Q)}$$

4. Calculate the estimated cost for a given quantity

$$Cost=\$100+\$10(18)=\boxed{\$280.00}$$

Question 4 (continued):

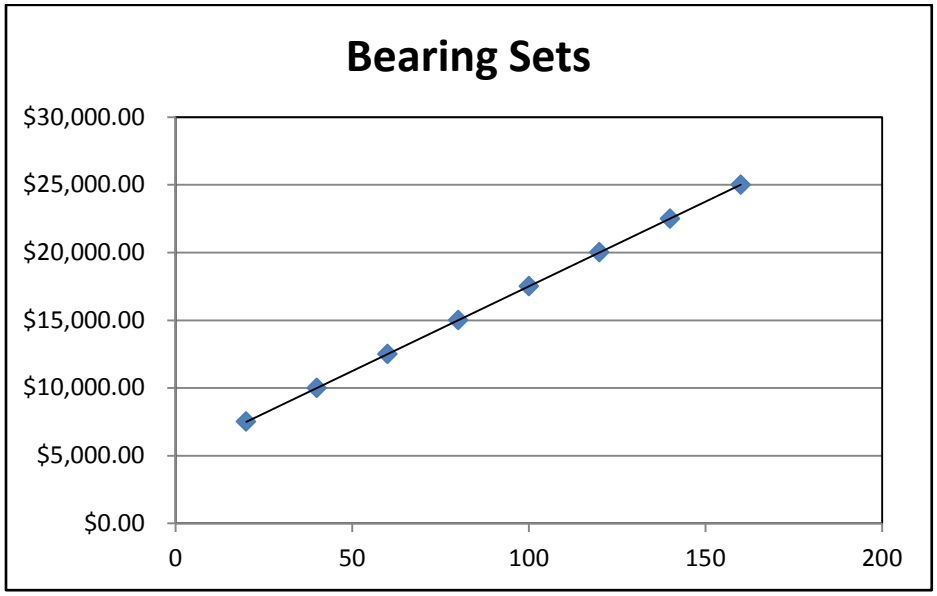
Based on this estimating equation, hypothetically, what is the cost to produce zero wiring harnesses? *$100. The contractor must pay its fixed costs even if zero units are produced. To calculate, insert "0" into the estimating equation. For a rough estimate, simply extend the estimating line all the way to the y-axis (where x = 0).*

Does this appear to be close to the estimate we figured simply by looking at the graph at the beginning of this lesson? *Yes! The graph estimate and the calculation should be close.*

Exercise Solutions (Cont'd)

Exercise 6.

1. Graph the data of Quantity in relation to Total Cost.



2. Calculate the variable cost:

$$V_U = \frac{C_2 - C_1}{Q_2 - Q_1}$$

$$\frac{\$10000 - \$7500}{40 \text{ units} - 20 \text{ units}} = \$125 \text{ per unit}$$

3. Using the data set and the variable cost, calculate the fixed cost element.

F = C - Vu(Q)
F = $10000 - ($125) (40)
F = $5000

4. Based on answers to questions 1 and 2, write the estimating line's equation!

C = F + Vu(Q)
C = $5000 + $125 (Quantity)

5. Based on the customer's request, what is the price estimate for 130 ball bearing sets:

C = $5000 + $125(130)
C = $21,250

6. How about the estimated price for 200 sets?

(Same estimating equation; now simply plug in 200 for Q)
C = $5000 + $125 (200)
C = $30,000

7. Based on your market research, what is the lowest price you would reasonably expect to pay?

Given market research data, you would expect contractors to charge a price which would enable them to cover all of their fixed costs, variable costs, and earn a reasonable profit. Thus, the first unit produced would be at least $5,125, plus profit (which covers all fixed and variable costs). We would expect 10 units to be $6,250 plus profit. While market research data is never a perfect predictive tool; if a proposed price is significantly lower than the data indicates, we must:

- *Be cautious—sometimes a low price is not "fair and reasonable"*
- *Conduct additional market research and fact-finding, considering within the guidelines set by TINA and FAR 15.402 and 15.403, as discussed in Unit 1 Lesson 2.*
- *Conduct additional market research and fact-finding before considering the offer to be unreasonably low—build an argument for your decision to either:*
  - *not award, because the offer initially appears to be "too low," a "low-ball," or a "buy-in," and the contractor has no logical explanation; or*
  - *award, even if the offer is lower than usual, based on your additional fact-finding*
- *Contractor should be able to explain how, at a significantly low price, he can still cover his fixed and variable costs. Answers could range from:*
  - *"I have more business than expected this year; all my fixed costs are covered; therefore, I can offer this price as a one-time good deal, and still recover all my costs and still make a profit."*
  - *"I just received a grant, which covered all my fixed and most of my variable costs for the year; therefore, I can offer this one-time good price, cover all my costs, and still make a profit."*
  - *"I just purchased new equipment, which I've amortized over 10 years. The new equipment has decreased my fixed costs, and significantly decreased my variable costs. Therefore, I intend to offer this new price to capture more market share. Soon, my competitors will also buy this new equipment, and I want customers to know me first."*

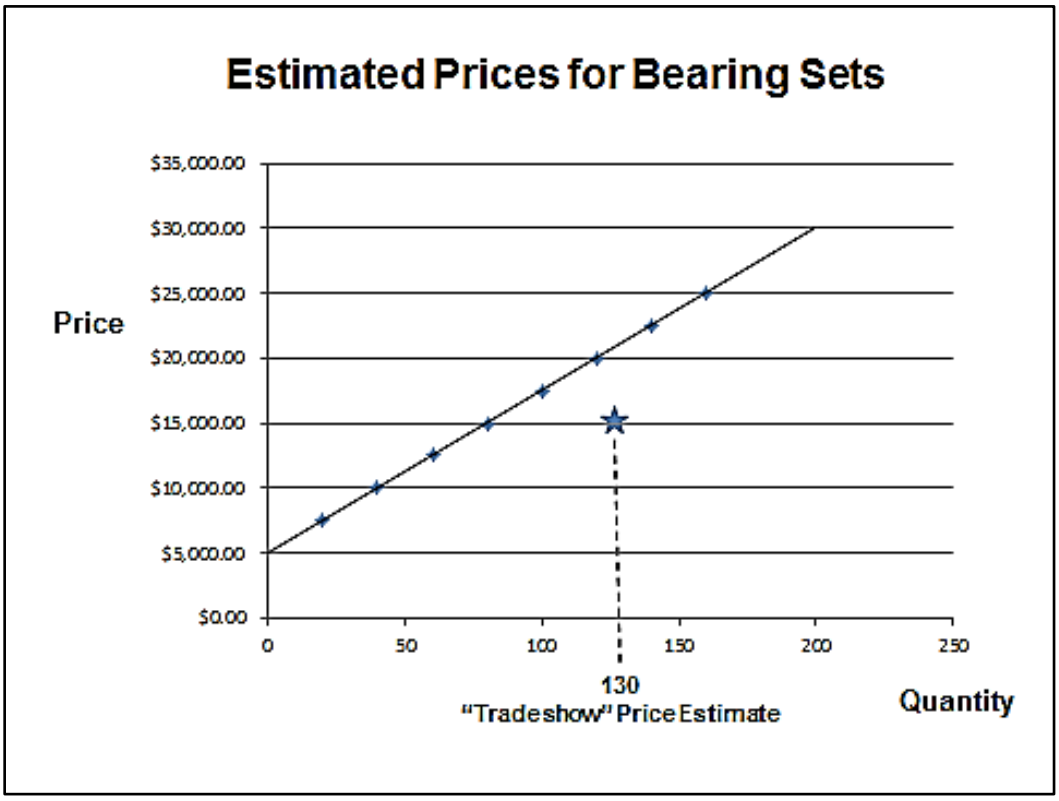
8a. After a recent, city-wide tradeshow, a small business representative visits your office, and says he can deliver 130 units for a total cost of $16,500. On the graph below, you compared this price with the rest of your market research data. Based on your market research data, and your cost-volume analysis, do you believe this is a reasonable offer? What questions would you ask?

See graph below, with this "Tradeshow Price" of 130 Units at Price of $16,500. Drive students to realize this is significantly lower than the market research data; but, is it reasonable? Learning point is, we would have to do fact-finding through additional market research. Good news for us here is, there is not a solicitation published yet, so fact-finding is much simpler!

Estimated price analysis:

- *As we calculated above, our estimated total cost for 130 units is $21,250.*
- *His offer of $16,500 is well below (about 22% less than) our estimate.*

- *How would we discern if this is a rational offer, rather than a "get the foot in the door," low-ball, irrational offer? After all, this offeror may have implemented newer, more efficient processes! Thus, some questions to ask:*
  - *<u>Have you sold at this price before</u>**?** <u>At a similar quantity, quality and delivery schedule</u>? <u>May I see records of previous sales history</u>? <u>Will it be valid next year</u>? <u>As a last resort, ask for cost information, to verify he can actually cover his costs</u>. Such questions follow the "order of preference" from our TINA lesson!*
  - *My market research indicates that appears to be an attractive price. Are you able to cover your fixed and variable costs?*



8b. The offeror responds to your questions, "I'm in a business cycle where I have received more orders that I expected on other contracts. Therefore, all of my fixed costs are covered for the rest of this year. For the rest of this year, I can offer you a better price than normal." What do you think?

- *By our estimating equation, if this offeror has the fixed costs covered through other contracts, his total costs will be $5,000 less than we'd normally expect: 130 units × $125 per unit = $16,250. At his proposed price of $16,500, the offeror is covering his variable costs, has all fixed costs covered, and is still earning $50 in profit.*

- *If the offeror can make the case he can cover his fixed and variable costs, the price can still be considered rational.*

- *If the offeror is not covering at least his variable costs, it may be an irrational,"low-ball," or "buy-in" offer. Remember, it is seldom a good idea to award a contract knowing the contractor is losing money! Such a scenario typically presents more risk than benefit.*

Discuss: "What if the offeror is deliberately planning to lose money, with hopes of getting his foot in the door with the Government customers?"

Contractors may do this (submit an apparent "low-ball"), but our job is to recognize those situations through this type of analysis, rather than simply not awarding based on a "gut feel" of an offer that appears to be too low. This is a logical follow on to the Lamm & Vose article about the different seller pricing strategies, and is also a relevant lead-in to Contribution Income…coming up in the next lesson.

Lesson 7 -- ELO 2.07 Through Cost-Volume Profit (CVP) analysis, recognize the nature of profit, revenue, and contribution income in calculating the contractor's "break-even" point

Introduction: From CVA to Cost-Volume-Profit (CVP)

Until now, we have only looked at the cost-volume relationship. Now, we are going to expand that relationship to consider the relationship between cost, volume, and **profit**. The revenue taken in by a firm is equal to cost plus profit.

That can be written: **Revenue = Total Cost + Profit**

We have already seen that total cost (C) is: $\mathbf{C = F + V_U\ (Q)}$

Using this information, we can rewrite the Revenue equation as: $\mathbf{Revenue = F + V_U\ (Q) + Profit}$

In the cost-volume-profit equation, **profit can be positive, negative, or zero**. If profit is negative, we normally refer to it as a loss. If profit is zero, the firm is breaking even, no profit or loss. If we let P stand for profit, we can write the equation:

$$\mathbf{Revenue = F + V_U\ (Q) + P}$$

Revenue is the product of two elements:

a. Revenue per unit, which is the selling price, represented by "Ru"; and
b. Quantity, or volume of units sold, represented by "Q"

The Revenue equation is written as follows:

$$\mathbf{Revenue = R_U\ (Q)}$$

If we assume that the firm makes all the units that it sells, and sells all the units that it makes, we can complete the CVP equation:

$$\mathbf{Ru\ (Q) = F + V_U(Q) + P}$$

Introducing Revenue and Profit

- Integrating our new terms:

  Revenue = Total cost + Profit

  R = F + Vu(Q) + Profit

- Finally, if we assume a firm sells all units produced, we can enhance our Revenue equation:

  Ru (Q) = F + Vu (Q) + Profit

Application of the Cost-Volume-Profit Equation

This equation and fundamental knowledge of a contractor's cost structure can provide you with extremely valuable information on the effect purchase decisions can have on a firm's profitability.

Using CVP Analysis to Estimate a Selling Price. The **Acme** firm prepared an offer for an indefinite quantity contract with the Government for a new product developed by the firm. Acme's goal is to sell 5,000 units, and earn $7,500 profit during this contract period. There are no other customers for the product. Given the following information, what is the selling price (R_U) Acme must charge in order to earn its profit goal?

Fixed Cost	=	$10,000
Variable Cost per Unit	=	$20
Contract Minimum Quantity	=	4,000 units
Contract Maximum Quantity	=	6,000 units
Firm's Best Estimate of Quantity	=	5,000 units
Target Profit	=	$7,500

Here are the steps to solve for Ru:

$$Ru(Q) = F + Vu(Q) + P$$

$$Ru(5{,}000) = \$10{,}000 + \$20(\$5{,}000) + \$7{,}500$$

$$Ru(5{,}000) = \$10{,}000 + \$100{,}000 + \$7{,}500$$

$$Ru\ (5{,}000) = \$117{,}500$$

$$\frac{Ru\ (5{,}000)}{5{,}000} = \frac{\$117{,}500}{5{,}000}$$

$$Ru = \$23.50$$

Therefore, in order to earn $7,500 profit on a production of 5,000 units, the selling price for each unit must be **$_____**.

Using CVP Analysis to Estimate Profit.

Potentially good news: it appears Acme will receive orders for 6,000 units. In the space below, using the CVP equation and Acme's data from above, calculate how much profit would Acme earn if it exceeded its sales goals and sold 6,000 units? ***(Hint: insert values for Ru, Q, F, and Vu, and solve for P)***

$$Ru(Q) = F + Vu(Q) + P$$

If Acme produced and sold 6,000 units at a selling price of $23.50, Acme would earn **$_____** profit. This is a **$_____** increase from the original $7,500 target profit. Note that Acme's profit would increase solely because sales were higher than estimated. This assumes the F and Vu relationship remain perfectly linear with an increase in Q. This is "linear" assumption will be used throughout CON 170. CON 270 will teach how to analyze non-linear relationships.

In addition, Acme's leaders were concerned about how profits would be affected if Acme sold only the minimum quantity of 4,000 units. In the space below, use the CVP equation and Acme's data from above to calculate how much profit Acme would earn if it only sold 4,000 units at a selling price of $23.50 per unit.

$$Ru(Q) = F + Vu(Q) + P$$

If the Acme sells 4,000 at $23.50 per unit, profit will be **$_____**. That is **$_____** less than the original $7,500 target profit. Note that Acme earns less profit because sales were lower than originally forecasted; however, Acme is still not "losing" money.

This leads us to another critical aspect of CVP: determining the "break-even point."

Using CVP Analysis to Calculate the Break-even Point

The "break-even point" is the quantity a firm must produce and sell in order to cover all of its fixed and variable costs, and begin to make a profit. When analyzing the risk of submitting a proposal to

an indefinite delivery contract, Acme's leaders assess the number of sales required for the firm to "break even"—ensuring they understand the quantity they must sell to cover their total cost, and begin to earn profit.

From Cost-Volume-Profit Analysis to the Break-even Point

- Break-even point: Quantity at which:
  - All fixed and variable costs are paid
  - Revenue equals, surpasses cost
  - Contractor begins earning profit
- Until contractor reaches break-even point, can not cover all costs, does not earn profit
- Why is the break even point important?
  - Answers the key question: "How many units do I have to sell before I actually earn profit?"
  - Enables an assessment of risk, and seller's pricing strategy

Break-even Point

- When calculating the break-even point:
  - Use the CVP equation, solve for "Q"
  - Solving for the Quantity where revenue equals total costs, and profit is set at zero:

$$\text{Revenue} = \text{Total Cost} + \text{Profit}$$

$$R = \text{Total Cost} + \$0$$

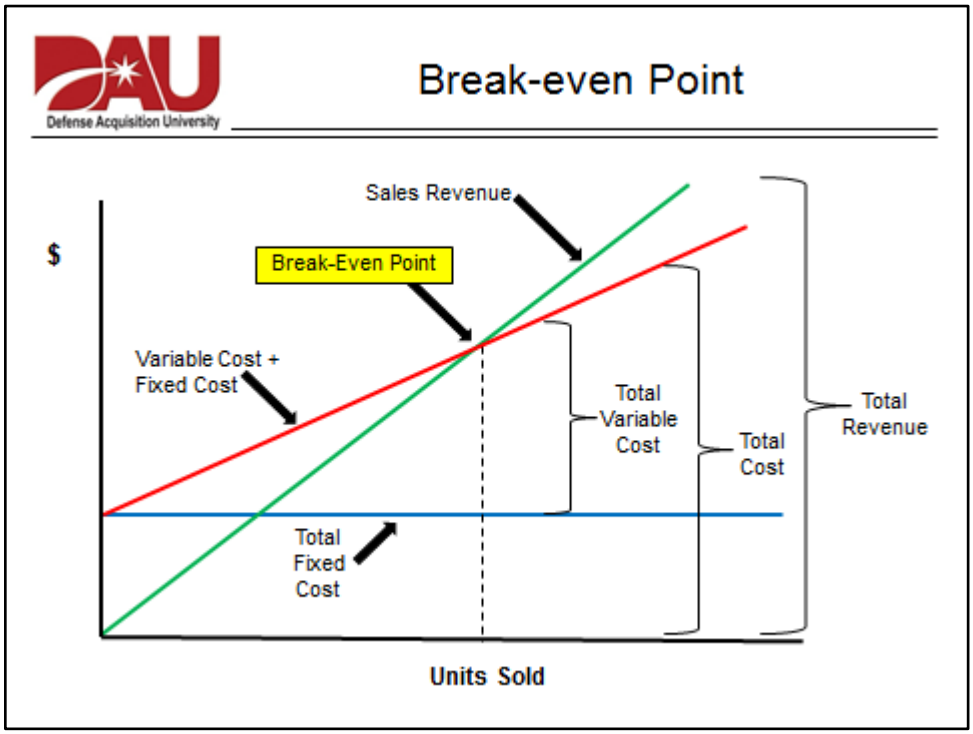
$$Ru\ (Q) = F + Vu\ (Q)$$

- Always always always "round up" Q to the next whole number—yes, even if the fraction is less than .5!!

Critical reminder: with regard to the break-even point, what is the last bullet on the slide above?

The graph below illustrates the concept of "Break-even Point." Notice the lines in the graph:

- Sales Revenue line represents Ru(Q): the unit selling price × quantity
- Total Cost line represents F + Vu(Q): fixed cost + (variable cost × quantity)
- These two lines intersect where Ru(Q) = F + Vu(Q), which is the break-even point!
- At the break-even point, the Sales Revenue (point to the line) is equal to the Total Cost (point to the "Variable Cost + Fixed Cost" line), and Profit = $0
- To the left of the break-even point, the contractor is operating at a "loss," for his revenue does not cover all fixed and variable costs.
- At the break-even point, the contractor's revenue finally covers all fixed and variable costs.
- To the right of the break-even point, the contractor has covered all fixed and variable costs, and begins to earn profit.
- Profit is represented by the wedge between the "Sales Revenue" line and the "Total Cost" line, to the right of the break-even point.



Check for Understanding

On the graph above, what is represented by the "wedge" under the Total Cost line, over the Sales Revenue line, to the left of the break-even point?

Where is profit depicted?

If an item has a low fixed cost, a low variable cost, and a high selling price, where on the graph would the breakeven point be…lower left or bottom right?

Comparing the total cost line of two different commodities, which would have a <u>steeper</u> slope, one with a higher variable cost, or lower variable cost?

To continue our Acme example, use the CVP equation to calculate the quantity of units sold (Q) to enable Acme to break-even.

Fixed Cost	=	$10,000
Variable Cost per Unit	=	$20
Selling Price	=	$23.50
Contract Minimum Quantity	=	4,000 units
Contract Maximum Quantity	=	6,000 units
Firm's Best Estimate of Quantity	=	5,000 units
Target Profit	=	$7,500

$$Ru(Q) = F + Vu(Q) + P$$

Break-even point is the Q where $Ru(Q) = F + Vu(Q) + P$, and we set P equal to $0

The calculations show that the firm would break even at _____ units. Because the firm could not sell .14 units, the firm must sell _____ units to assure that all costs are covered. To verify:

<u>Selling 2857 units</u>

Revenue = Total Cost + Profit

$$Ru(Q) = F + Vu(Q) + P$$

$$23.50(2857) = \$10{,}000 + 20(2857) + 0$$

$$\$67{,}139.50 = \$67{,}140.00$$

$$< \$.50$$

Revenue is $0.50 less than costs.
Acme has <u>not</u> reached "break-even" point.

<u>Selling 2858 units</u>

Revenue = Total Cost + Profit

$$Ru(Q) = F + Vu(Q) + P$$

$$23.50(2858) = \$10{,}000 + 20(2858) + 0$$

$$\$67{,}163.00 = \$67{,}160.00$$

$$> \$3.00$$

Revenue is $3.00 more than all costs.
<u>Acme reached "break-even" point.</u>

Example 1. Given the following information for an offeror's proposal for wiz-bangs, calculate the break-even point.

Fixed Costs:	$15,000
Variable Cost per Unit	$30.00
Selling Price	$33.08
Maximum Capacity	8,000 units

Remember, the break-even point is the quantity of units sold when the firm finally covers all of its fixed and variable costs, and begins to earn profit. Therefore, when calculating the break-even point, we use the CVP equation, and solve for "Q" where "P" equals zero.

CVP equation: $Ru(Q) = F + Vu(Q) + P$

Then, set P = 0, and solve for Q: $Ru(Q) = F + Vu(Q)$

Break-even Capacity

With an understanding of the break-even point, it is also important to understand how the contractor's break-even point compares to its overall production capacity. In the example above, we see the contractor must produce and sell at least 4,871 units to break even, and finally begin to earn profit. Is this production output within his production capacity? In this scenario, we see the contractor's maximum capacity is 8,000 units. Therefore, we have some assurance the contractor has a reasonable sound production plan, for he breaks even well within his maxium capacity.

Consider a contractor who's break-even point is 5,000 units, but his maximum capacity is 4,500 units. In this case, the contractor can not even produce and sell enough units to cover his total costs! Without a robust explanation, an award to this contractor would present unacceptably high risk.

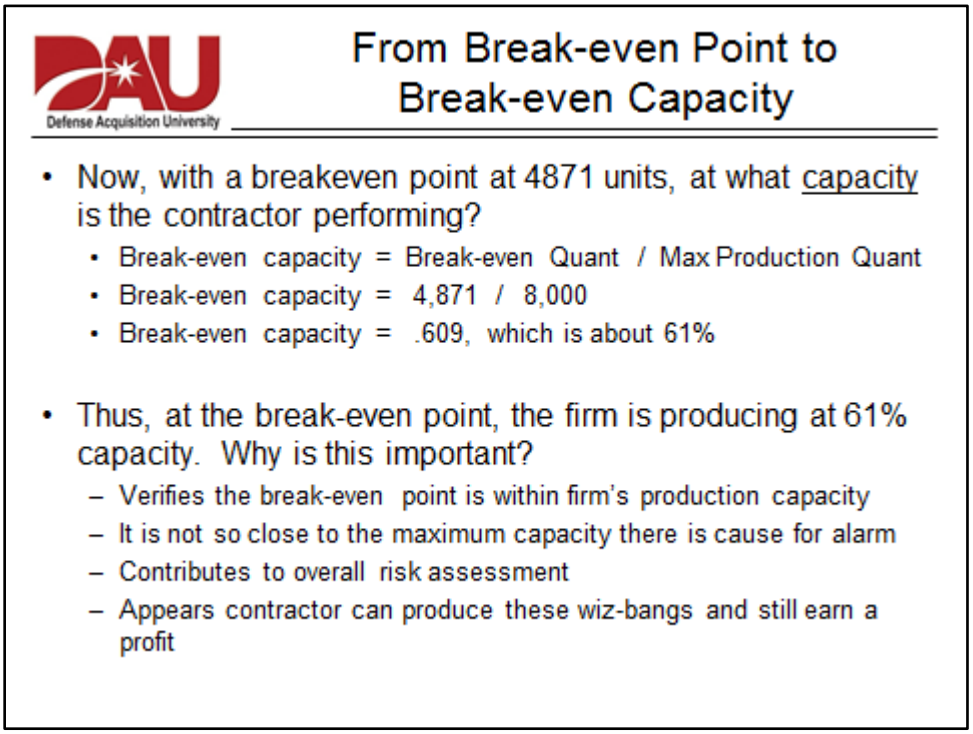
The break-even capacity is stated as a percentage. To calculate Break-even Capacity, use the simple ratio:

Break-even Capacity = Break-even Quantity / Maximum Production Quantity

From Example 1 above, the firm's break-even point was 4,871 units, and their maximum capacity (or their maximum production quantity) is 8,000 units. What is their "break-even capacity?"

Break-even Capacity = Break-even Quantity / Maximum Production Quantity

This means, at the point where the contractor breaks-even, he is operating at 61% of his capacity.



Check for understanding:

What is the ratio for calculating "break-even capacity?"

What is the purpose of understanding the break-even capacity?

State the risk of the following offers as low, high, or unacceptably high:

- Offer 1's break-even capacity is 30%.
- Offer 2's break-even capacity is 99%
- Offer 3's break-even capacity is 120%

Break-even Point and Contribution Income

Review and discussion:

- What are the key elements that impact the break-even point?
  - Fixed costs, variable costs, selling price
  - These costs are influenced by market variables
    - Shift in market supply and demand
    - Overall health of the economy
    - Excess capacity
    - Learning curve, capital investment
    - Accounting for fixed costs
- Brings us to final topic in Lesson 7…
  …Contribution Income!

Contribution Income

The difference between revenue and variable cost is contribution income (CI). The term contribution income comes from the contribution made to covering fixed costs and profit. If contribution income is positive, increasing sales will increase profits or reduce losses. If contribution income is negative, increasing sales will reduce profits or create greater losses.

$$\text{Contribution Income} = \text{Revenue} - \text{Variable Cost}$$

$$CI = R_U(Q) - V_U(Q) \quad \text{or}$$
$$CI = (R_U - V_U)\,Q$$

Knowledge of a contractor's cost structure and contribution income can be valuable in analysis of proposed costs.

Contribution Income

- In general:
  - Revenue beyond variable costs is "contributed" to fixed costs
  - Revenue beyond variable and fixed costs is "contributed" to profit
- Contribution Income (CI): Revenue minus Variable Cost

$$CI = R - VC$$
$$CI = Ru(Q) - Vu(Q)$$
$$CI = (Ru - Vu)(Q)$$

- CI can be calculated on a per-unit level, as well as a total quantity level (see next slide)

Contribution Income Example. In evaluating an offeror's proposal for 500 units at $900 each, your analysis reveals the following cost structure:

Fixed Cost	= $100,000
Variable Cost per Unit	= $1,000

How would this affect your analysis of contract risk?

$$CI = (R_U - V_U)\ Q$$
$$CI = (\$900 - \$1{,}000)\ (500)$$
$$CI = (-\$100)\ (500)$$
$$CI = -\$50{,}000$$

The contribution income from the sale is a <u>negative</u> $50,000. The firm would be substantially worse off for having made the sale. Unless the firm can offer a positive rationale for such a pricing decision, such as fixed cost has already been covered under a previous contract, you must consider pricing as an important factor as you analyze the risk of contract performance.

In Example 1 above, we were analyzing the production of wiz-bangs, where:

Fixed Costs:	$15,000
Variable Cost per Unit	$30.00
Selling Price	$33.08

In the space below, calculate the CI per unit:

Next, calculate the total CI for 6,500 units:

Another CI Example: in a competitive environment, a firm has offered to sell the Government 4,500 wiz-bangs for $25 each. Your assessment reveals:

Fixed Costs:	$15,000
Variable Cost per Unit	$30.00
Selling Price	$25.00

In the space below, calculate the CI per unit:

Next, calculate the total CI for 4,500 units:

Discussion Questions:

Is this offer to sell each wiz-bang for $25 rational or irrational?

What is the lowest price you would be willing to pay?

Questions to Consider in CVP Analysis - As you perform price/cost analysis, consider the issues and concerns identified in this section, whenever you use cost-volume-profit analysis concepts.

Has the contractor's cost structure changed substantially? CVP analysis assumes that the period covered by the analysis is too short to permit facilities expansion or contraction or other changes that might affect overall pricing relationships. If the contractor has substantially changed its cost structure, your ability to use CVP analysis may be limited. Examples of possible changes include:

- Downsizing to reduce fixed costs.
- Increased investment in automated equipment to reduce variable costs of labor and material.

Is the straight-line assumption reasonable? CON 170 assumes the CVP relationship is a straight-line (linear) relationship. In the "real world," market research will reveal the CVP relationship is not truly linear. Instead, CVP data points are scattered with variance over and above a general trend line. In CON 270, we will learn to build pricing estimates by analyzing variance and trend-lines.

Are current volume estimates within the relevant range of available data? If the current volume of goods or services we are acquiring is substantially higher or lower than the volumes used to develop the CVP equation, our estimating results may be unreliable. At higher or lower volumes, the contractor would likely be using different processes under a different cost structure, which would significantly change the CVP elements.

In-Class Exercises

Exercise 1. Lester's Lawn Mowing. As a 13-year old, Lester has a goal to earn enough money to attend college. His goal is to earn $1,250 of profit each summer until he graduates from high school. After some practice, he has developed skill in mowing lawns. With some guidance from his dad, Lester purchased a lawn-mower, a trimmer, and a broom for a total of $250. With gas and expendables, his variable cost per lawn is $10. After doing market research on the prices of lawn service, he intends to charge $30 per lawn.

1. How many lawns would he have to mow in order to earn this year's goal of $1,250?

Here are the necessary elements for solving this problem:
Fixed costs (F): *$*_____
Variable costs, Vu(Q): *$*_____ *per lawn*
Profit Goal (P): *$*_____
Selling price or Revenue per lawn, Ru(Q): *$*_____
Needs to mow _____ lawns to earn the profit goal?

2. Assuming the weather permits 15 weeks of lawn-mowing per year, and Lester can mow a maximum of 6 lawns per week. At what capacity is Lester operating in order to earn his profit goal? How much mowing risk do you believe Lester is managing to achieve his goal?

3. How would you describe the risk if he needed to mow 90 lawns to earn his profit goal?

4. Under his pricing arrangement, what is the "Contribution Income" per unit?

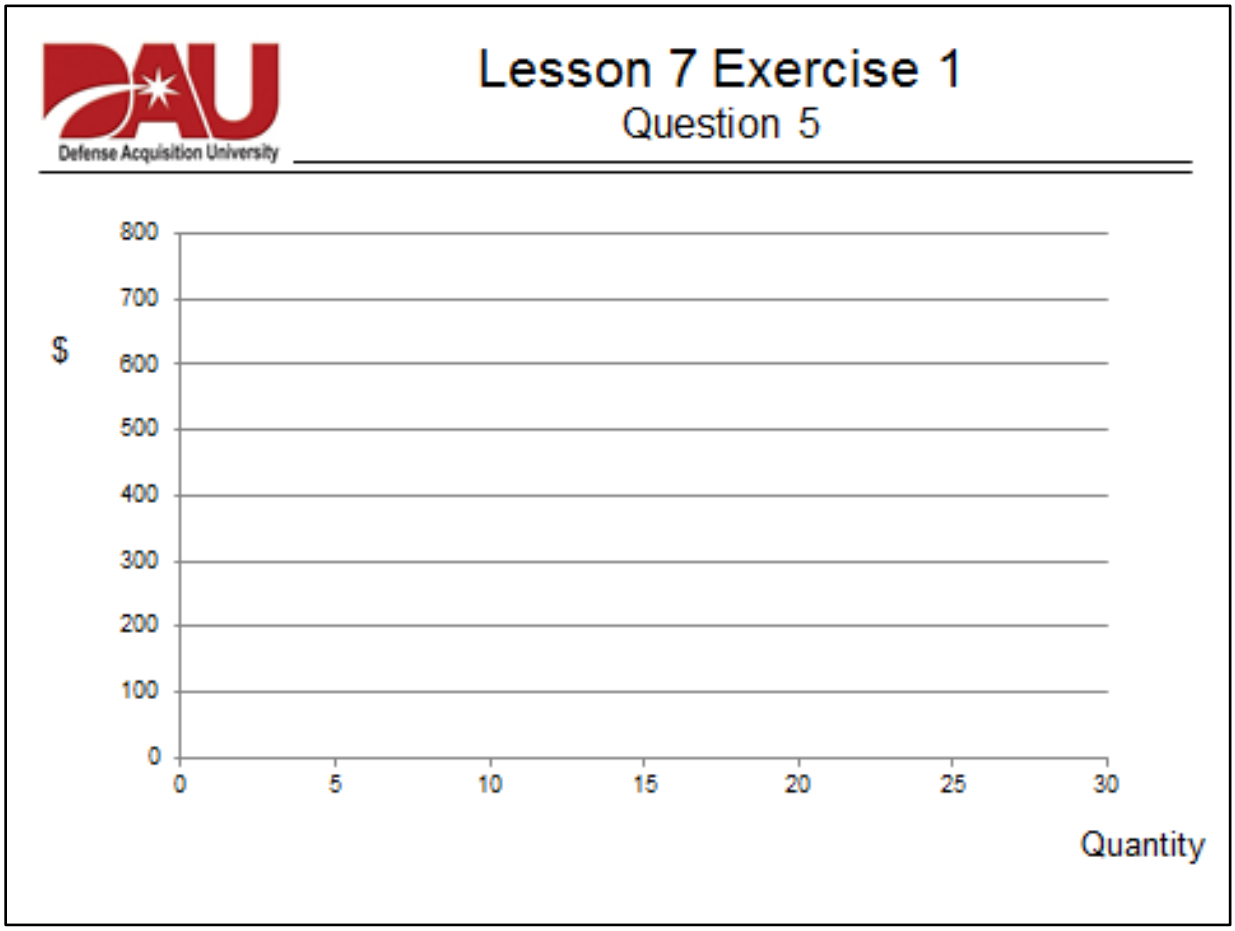
What is his total Contribution Income for 75 lawns?

5. Given Lester's fixed and variable costs, and his anticipated sales revenue, draw a graph to <u>estimate</u> the number of lawns Lester must mow to "break even." Remember:

- **Total cost line is graphed with C = F + Vu(Q);**
- **Sales revenue is graphed with R = Ru(Q)**

(Hint: use this framework to establish points for the Sales Revenue and Total Cost Lines)

Total Cost Line		**Sales Revenue Line**	
Q	Total Cost	Q	**Ru(Q)**



6. Given Lester's fixed and variable costs, and his anticipated sales revenue, <u>calculate</u> the breakeven point. Is your calculation consistent with your graph above? (Remember to "round-up")

7. Assuming the weather permits 15 weeks of lawn-mowing, Lester can mow a maximum of 6 lawns per week. At what capacity is Lester operating at his <u>break-even point</u>? Does this appear to be a risky venture?

Exercise 2. Jill's invention. Jill has figured out how to build her own version of an iPod™ player which also dispenses Pez™ candy. Her goal is to earn $25,000. She believes she can build and sell 5,000 units. Her fixed costs are $60,000, variable cost are $20 per unit.

1. To earn her profit goal, how much must she charge for each unit, assuming she sells all she makes?

2. Under this pricing arrangement, what is Jill's contribution income per unit?

What is her total contribution income at 5,000 units?

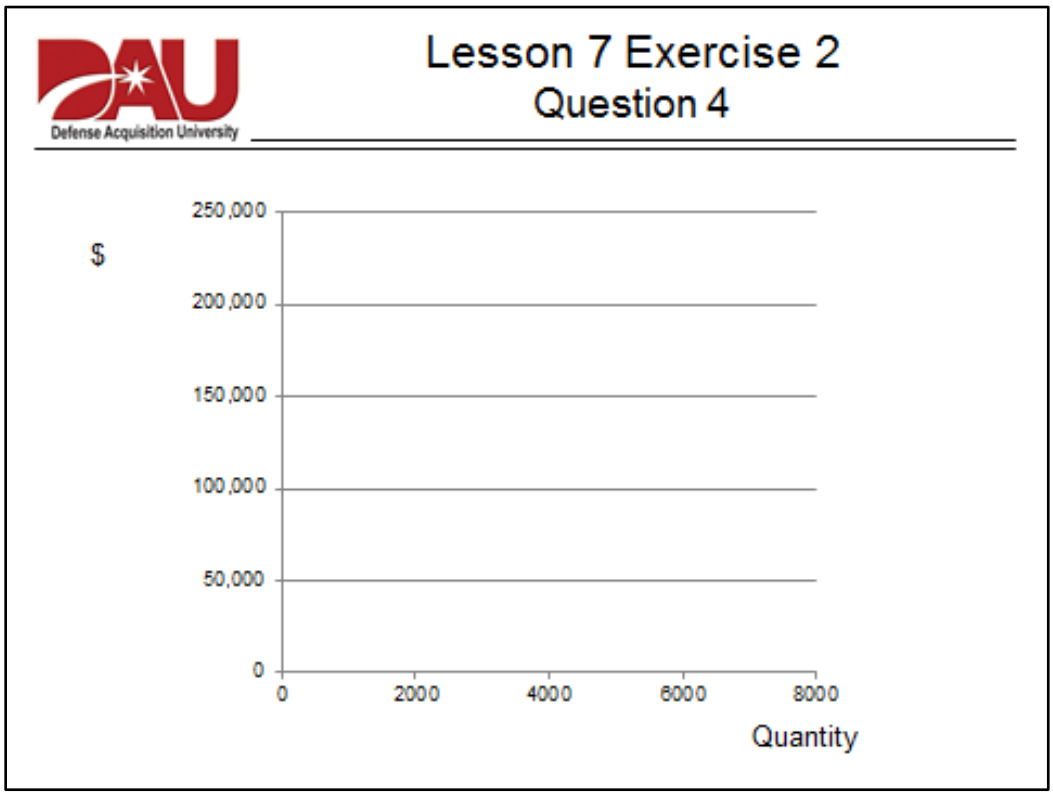
3. Jill acquires equipment to deliver a maximum production capacity of 6,000 units. At what capacity will she be producing to earn her profit goal?

4. Given her fixed and variable costs, and her anticipated sales revenue, draw a graph in the space below to <u>estimate</u> the number of units she must sell "break even." Remember:

- **Total cost line is graphed with C = F + Vu(Q);**
- **Sales revenue is graphed with R = Ru(Q)**

(Hint: use this framework to establish points for the Sales Revenue and Total Cost Lines)

Total Cost Line		**Sales Revenue Line**	
Q	Total Cost	Q	**Ru(Q)**



5. Under this pricing arrangement, <u>calculate</u> Jill's breakeven point (remember to "round up"). Is your calculated answer consistent with your estimate from the graph?

6. At what capacity is she producing at the break-even point?

7. At Jill's birthday party, she announces to her guests that she will sell this initial production run of 5,000 units for $24 per unit. Is she off her rocker? (i.e., is this a rational strategy, or irrational?). ***Hint: calculate her contribution income per unit, and total.***

8. Sensing her guests' uncertainty regarding her contribution income, Jill informs everyone, ***"For my birthday, a venture capitalist paid my $60,000 in fixed costs. With less risk, I was willing to lower my profit goal to $20,000."*** **Given this new information, does she have a rational strategy?**

Exercise 3. Andrew the Aviator. (To be completed outside of class. Answers provided in back of lesson). Andrew has a goal to save enough money to pay for flying lessons. His goal is to earn $3,000 profit in one year. After some practice, he developed skill in drawing caricatures of CON 170 professors. He purchased an easel, markers, and art paper for a total of $300. With art supplies and labor, his variable cost per drawing is $4. After doing market research, he believes he can sell each caricature for $25 each.

1. How many caricatures would he have to draw in order to earn his goal of $3,000? (remember to "round up")

Fixed Costs, F: $_____
Variable Cost per drawing, Vu(Q): $_____ per unit
Profit Goal, P: $_____
Selling Price, Ru(Q): $_____
Andrew needs to sell _____ caricatures to earn his profit goal of $3,000.

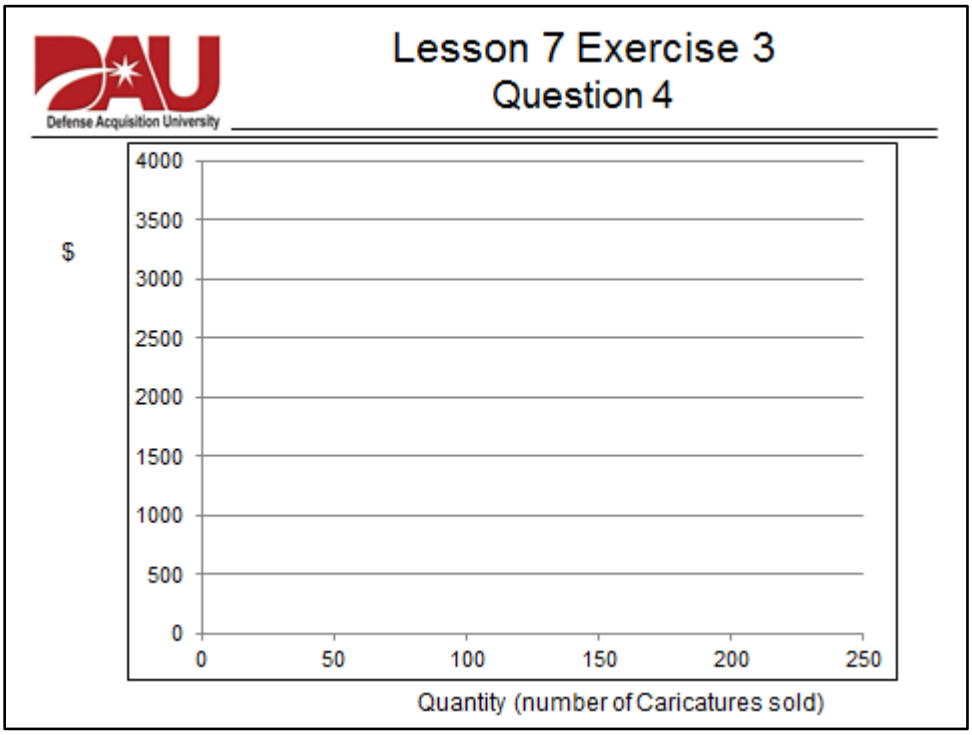
2. Assuming Andrew's schedule permits a maximum of 35 weeks when he can draw and sell 16 caricatures per week, at what capacity is Andrew operating in order to earn his profit goal? How much risk do you believe Andrew is managing to achieve his goal?

3. Under this pricing arrangement, what is Andrew's Contribution Income per drawing?

4. Given Andrew's fixed and variable costs, and his anticipated sales revenue, draw a graph to estimate the number of caricatures Andrew must draw and sell to "break even."

(Hint: use this framework to establish points for the Sales Revenue and Total Cost Lines)

Total Cost Line		**Sales Revenue Line**	
Q	Total Cost	Q	**Ru(Q)**



5. Given Andrew's fixed and variable costs, and his anticipated sales revenue, calculate the break-even point. Verify your calculations by comparing your answer to the graph above.

6. Assuming 35 weeks of drawing and selling 16 caricatures per week, at what capacity is Andrew operating at the <u>break-even point</u>? Does this appear to be a risky venture?

Exercise 3 Solution

1. How many caricatures would he have to draw in order to earn his goal of $3,000?

Fixed Costs, F: **$300**
Variable Cost per drawing, Vu(Q): **$4**
Profit Goal, P: **$3,000**
Selling Price, Ru(Q): **$25**
Andrew needs to sell **158** caricatures to earn his profit goal of $3,000.

To reach his profit goal, use the CVP equation, and solve for Q where profit is $3,000.

Ru(Q) = F + Vu(Q) + P
$25(Q) = $300 + $4(Q) + $3,000
21(Q) = $3,300
Q = 157.14

*Remember to always round up when calculating break-even and profit goal calculations. Andrew will not reach his profit goal until he sells his **158th** drawing.*

2. Assuming Andrew's schedule permits a maximum of 35 weeks when he can draw and sell 16 caricatures per week, at what capacity is Andrew operating in order to earn his profit goal? How much risk do you believe Andrew is managing to achieve his goal?

35 weeks X 16 drawings per week = 560 total drawings, which is his max capacity

3. Under this pricing arrangement, what is Andrew's Contribution Income per drawing?

CI = Ru(Q) - Vu(Q)
Or, CI = (Ru - Vu)(Q)
CI = ($25 - $4)(1)
CI = $21 per unit

4. Given Andrew's fixed and variable costs, and his anticipated sales revenue, draw a graph to estimate the number of caricatures Andrew must draw and sell to "break even."

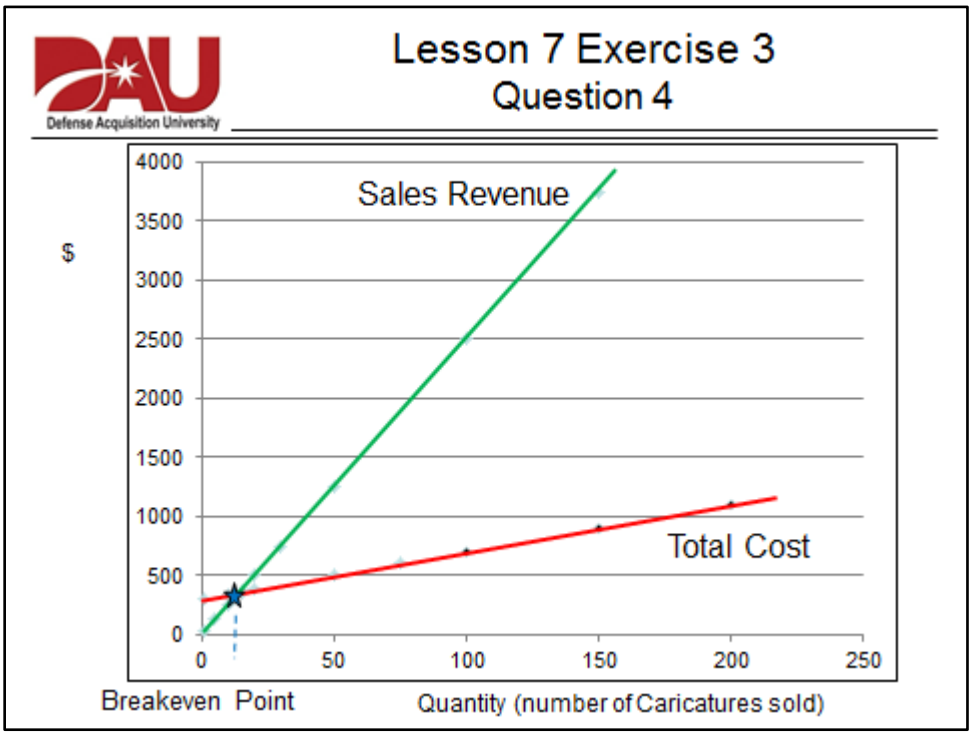
(Hint: use this framework to establish points for the Sales Revenue and Total Cost Lines)

Total Cost Line		**Sales Revenue Line**	
Q	Total Cost	Q	**Ru(Q)**
10	*340*	*10*	*250*
15	*360*	*15*	*375*
50	*500*	*50*	*1250*
100	*700*	*100*	*2500*
150	*900*	*150*	*3750*
200	*1100*	*200*	*5000*

To establish points for our Total Cost and Sales Revenue lines, select at least three points for each line, as shown above. The points given above were selected at random, using "easy to calculate" whole numbers.

***First**: given graph below, plot the Total Cost line (which is F + Vu(Q)).*

***Second**: plot the Sales Revenue line (which is Ru(Q) on the y-axis and Q on the x-axis)*
***Finally**, note where the lines intersect—the value for Q at this intersection point is the "break-even point." This appears to be where Q is more than 10, but less than 20. This intersection point represents where sales revenue intersects and surpasses the total costs to draw and sell the caricatures, and Andrew begins to earn profit.*



5. Given Andrew's fixed and variable costs, and his anticipated sales revenue, calculate the break-even point. Verify your calculations by comparing your answer to the graph above.

Break-even point is the Q where Ru(Q) = F + Vu(Q) + P, and we set P equal to $0
Therfore: *$25(Q) = $300 + $4(Q)*
$21(Q) = $300
Q = 14.29 drawings to break even.
However, because we can not draw and sell ".29" drawings, we must round up!!
` *Thus, Andrew will cover all of his costs, and begin to earn profit when he sells his **15th** drawing!*

6. Assuming 35 weeks of drawing and selling 16 caricatures per week, at what capacity is Andrew operating at the break-even point? Does this appear to be a risky venture?

Break-even quantity of 15 / Max capacity of 560 = .027, which is 2.7%.
Thus, Andrew will be operating at 2.7% of his maximum capacity to break-even.
This is low risk, for he can break even well within his maximum capacity.

Additional Example (Optional—not tested in CON 170--looking ahead to CON 270):

Consider the following. Our maintenance customer asked us to acquire 200 widgets. We've worked together to accomplish market research.

Additional Example
(looking ahead to CON 270)

-Your customer needs you to buy 200 widgets.

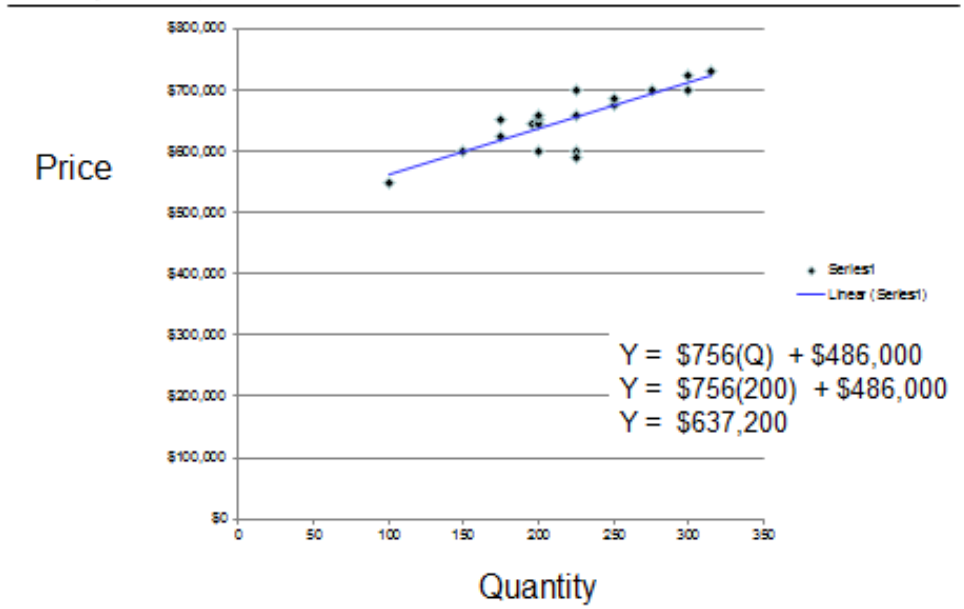
- You accomplish market research and found the following price history.

- So, given this data, what would I estimate the price of 200 units for my customer?

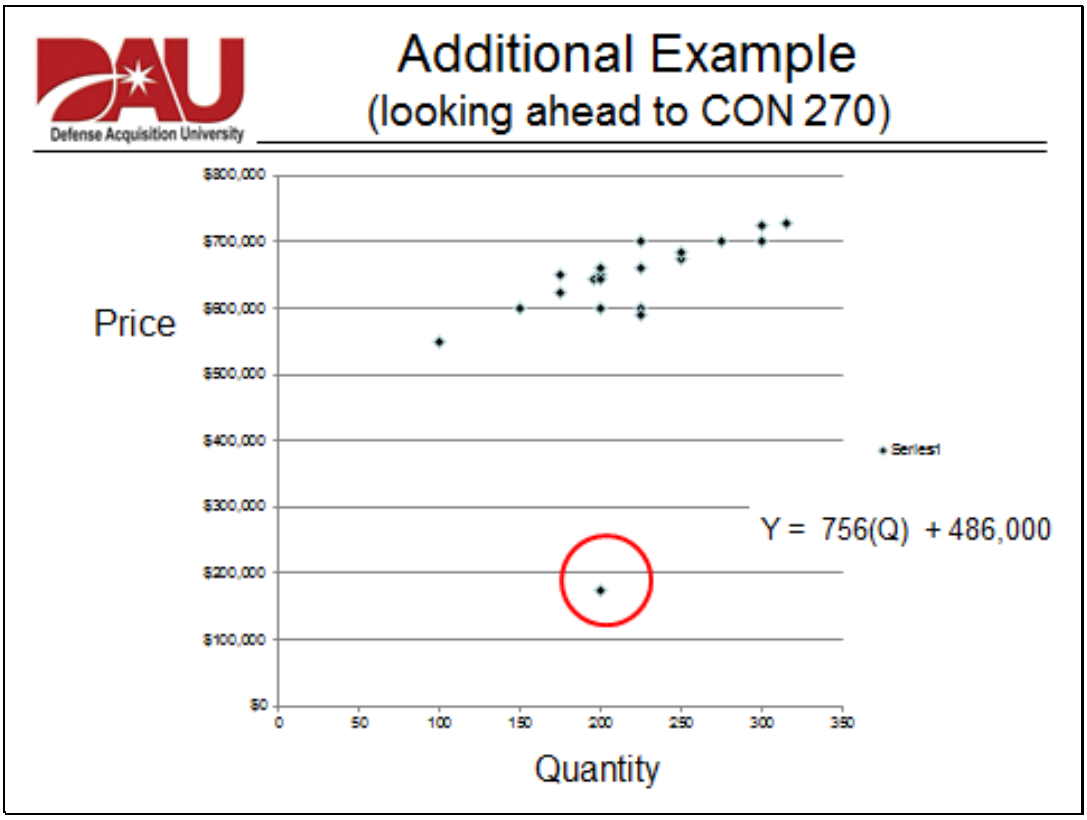
Quantity	Price
100	$550,000
150	$600,000
175	$625,000
175	$650,000
195	$645,000
200	$650,000
200	$645,000
200	$660,000
200	$600,000
225	$660,000
225	$600,000
225	$700,000
225	$590,000
250	$675,000
250	$685,000
275	$700,000
300	$725,000
300	$700,000
315	$730,000

In building the price estimate with Microsoft Excel's "scatterplot" tool, we see the trend line among the data points, as well as the estimating equation. With this equation, we can estimate the price for 200 units to be $637,200.

Additional Example
(looking ahead to CON 270)



During a pre-proposal conference, one firm indicates they will sell us 200 units for $925 each, for a total price of $185,000. Through the discussion, it is clear the firm understands the requirement, and is considering terms and conditions similar to previous purchases. Clearly, this price is significantly lower than we would expect. Based on the graph below, do you have any concerns?



What should you do to determine if this is a rational offer under a market penetration type strategy, or an irrational offer under a "buy-in" strategy?

- **Based on your price history, and the estimating line:**
  - *What are the fixed costs:* $_____
  - *What are the variable costs:* $_____
  - *Selling price:* $_____

- **What is the contribution income:**